

Lecture 5 – Search Based Software Testing (SBST)

AAA705: Software Testing and Quality Assurance

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2024 Spring

Sometimes called **structural testing** because it uses the **internal structure** of the program to derive test cases.

- **Coverage Criteria**

- The adequacy of a test suite is measured in terms of the **coverage** of the program's internal structure.

- **Search Based Software Testing (SBST)**

- A technique that uses **meta-heuristic search** algorithms to maximize/minimize a certain **fitness function**.

- **Dynamic Symbolic Execution (DSE)**

- A technique that systematically explores the input space using **symbolic execution** with **dynamic analysis**.

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Let's focus on the **SBST** in this lecture, and start from **search-based software engineering (SBSE)**!

1. Search Based Software Engineering (SBSE)
2. Fitness Landscape
3. Local Search
 - Hill Climbing
 - Simulated Annealing
 - Tabu Search
4. Genetic Algorithms
 - Selection Strategies
 - Crossover Operators
 - Mutation Operators
5. Bio-inspired Algorithms
 - Particle Swarm Optimization (PSO)
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 - Alternating Variable Method (AVM)

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- Two major conferences (ICSE and ESEC/FSE) now tend to have whole sessions dedicated to SBSE.
- Dedicated international conference (e.g., SSBSE) and many other workshops.

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- **Approximate** and usually non-deterministic

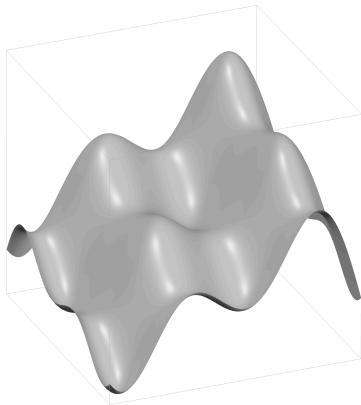
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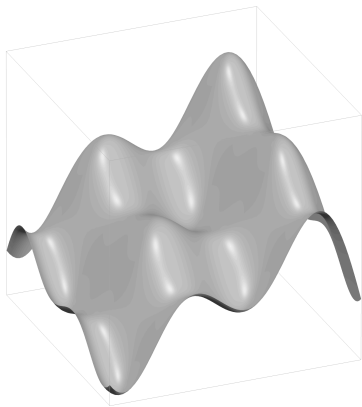
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- Strategies that **guide** the search process to find **acceptable solutions**
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- **General** and not problem-specific
- **Iterative** improvement by **exploring** the search space

How to find the **best** or at least an **acceptable** solution?



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Try and automatically **learn** from the **experience** for the next **trial**.

- **Representation** – **What** are we going to try this time?

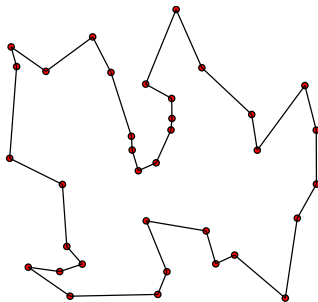
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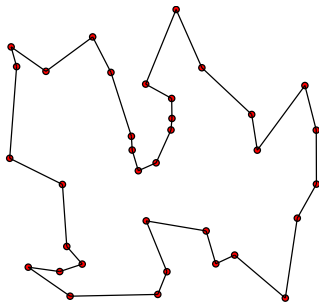
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- Constraints, etc.

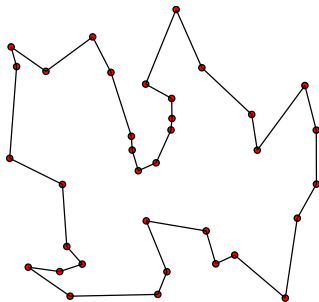
Example: Travelling Salesman Problem (TSP)



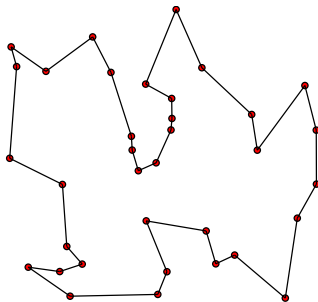
- Assume that you are a salesman.



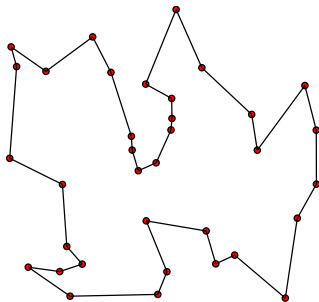
- Assume that you are a salesman.
- You want to **visit all** the cities and **return** to the starting city with the **minimum cost** (e.g., distance, time, etc.).



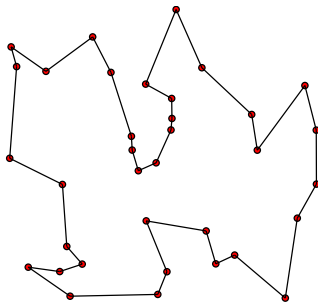
- Assume that you are a salesman.
- You want to **visit all** the cities and **return** to the starting city with the **minimum cost** (e.g., distance, time, etc.).
- Unfortunately, the TSP is a **NP-hard** problem. It means that there is **no known algorithm** that can solve it in **polynomial time**.



- **Representation:** A sequence of cities



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- **Representation:** A sequence of cities
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- **Fitness Function:** Total distance

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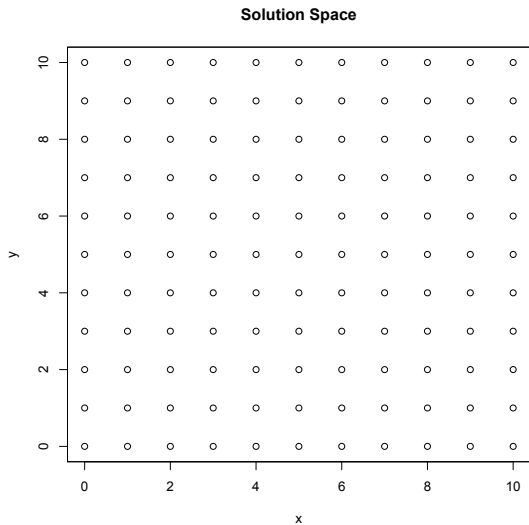
- **Exploitation:** If we have found a good solution, we should try to search around it or do something similar.
- **Exploration:** Unexplored search space may contain **much better** solutions.
- How to **balance** these two is a **key** to the success of SBSE.

- **Fitness Landscape**
- **Local Search**
- **Genetic Algorithms**
- **Bio-inspired Algorithms**

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Fitness Landscape

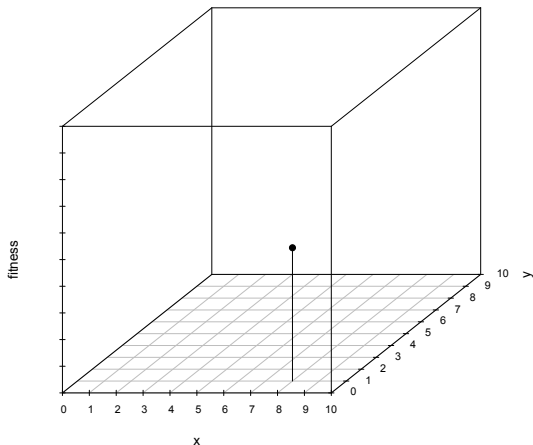
Let's consider a fake problem: Find the pair (x, y) such that $x + y = 10$ for $0 \leq x \leq 10$ and $0 \leq y \leq 10$.



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A single point in fitness landscape

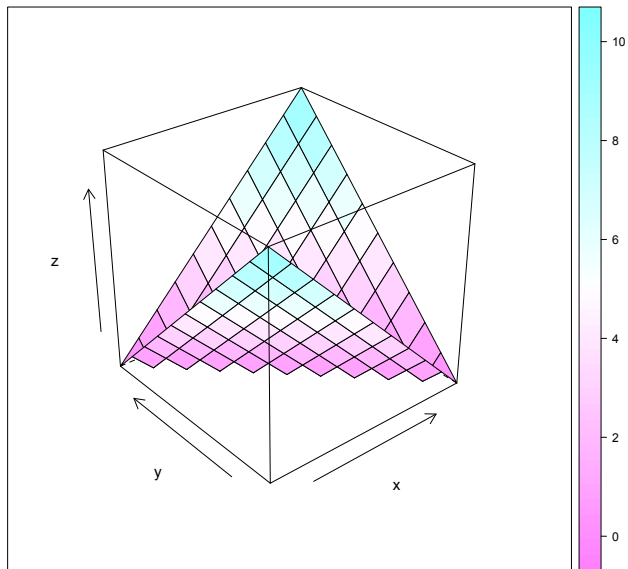


- For each **representation** (x, y) , how to know **how good** it is?

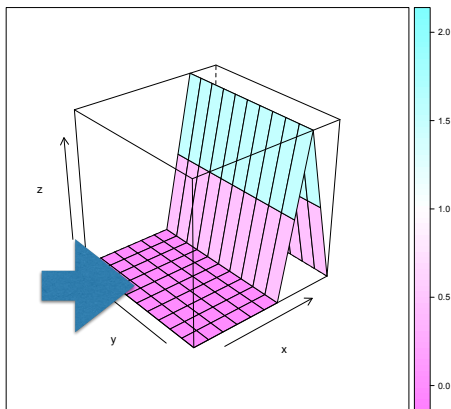
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- We need to solve the problem $x + y = 10$.
- We can **change** the problem into a **minimization** problem:

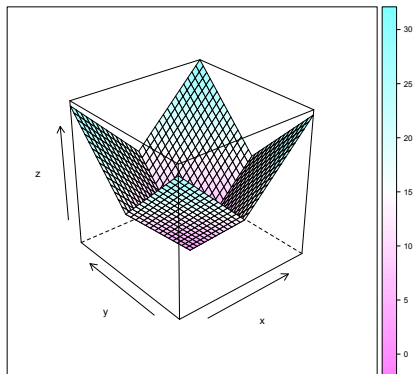
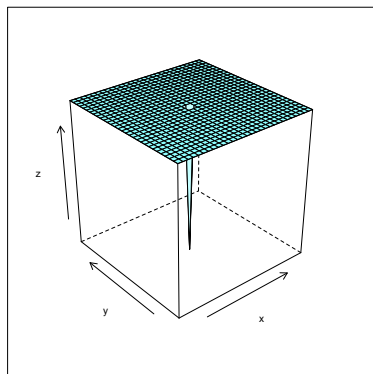
$$f(x, y) = |10 - (x + y)|$$



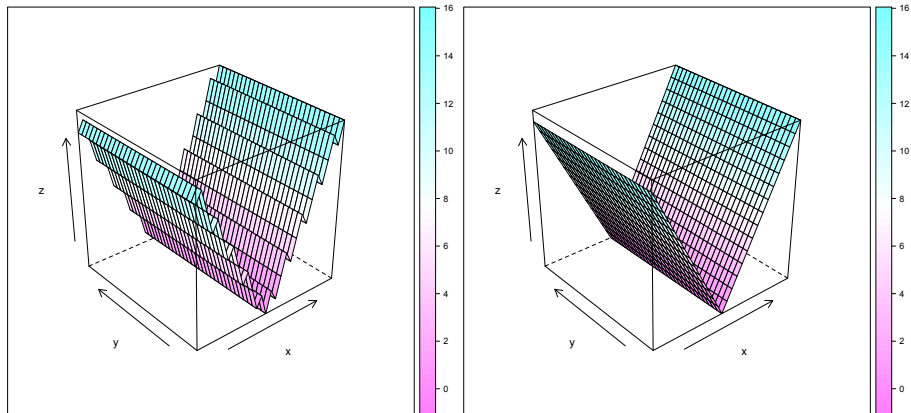
It is difficult to escape from the large and flat region (i.e., **plateau**) in the fitness landscape



If the fitness landscape has a small region of high fitness surrounded by a large region of low fitness, it is called a **needle in a haystack**, and it is the worst case for search algorithms. We need to find a way to change the landscape into a more favorable one.



If the fitness landscape has many local optima, it is called a **rugged** landscape. In this case, the search algorithm may get stuck in one of the many local optima and fail to find the global optimum.



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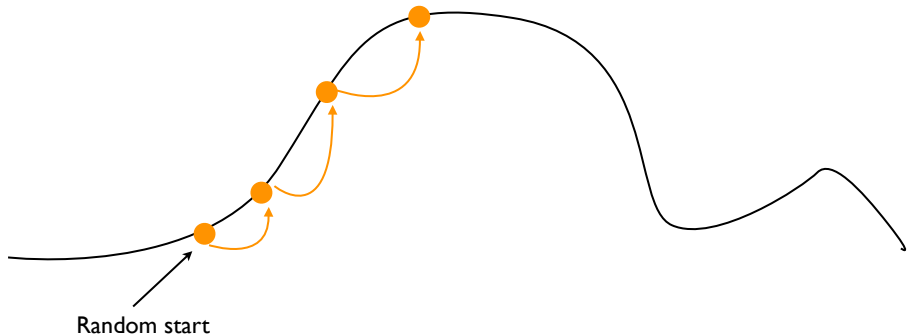
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- **Move** to one of **better** solutions according to the fitness function.
- **Repeat** the process until **no better solution** is found.



The most popular local search algorithm is the **hill climbing** algorithm with the **steepest ascent** strategy.

HILLCLIMBING()

- (1) $climb \leftarrow True$
- (2) $s \leftarrow GETRANDOM()$
- (3) **while** $climb$
- (4) $N \leftarrow GETNEIGHBOURS(s)$
- (5) $climb \leftarrow False$
- (6) **foreach** $n \in N$
- (7) **if** $FITNESS(n) > FITNESS(s)$
- (8) $climb \leftarrow True$
- (9) $s \leftarrow n$
- (10) **return** s

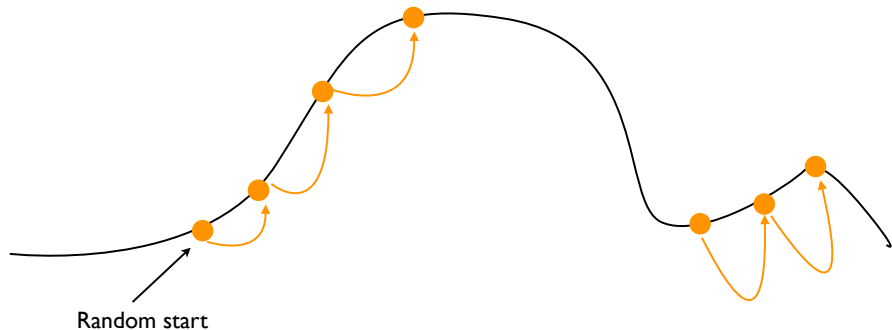
One of variations of the hill climbing algorithm is the **first ascent** strategy by selecting the first better solution.

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(5)     climb ← False
(6)     foreach n ∈ N
(7)       if FITNESS(n) > FITNESS(s)
(8)         climb ← True
(9)         s ← n
(10)      break
(11)  return s
```


Or, we can **randomly** select a solution among the better neighboring solutions in the hill climbing algorithm.

```
HILLCLIMBING()
(1)    $s \leftarrow \text{GETRANDOM}()$ 
(2)   while True
(3)      $N \leftarrow \text{GETNEIGHBOURS}(s)$ 
(4)      $N' \leftarrow \{n \in N \mid \text{FITNESS}(n) > \text{FITNESS}(s)\}$ 
(5)     if  $|N'| > 0$ 
(6)        $s \leftarrow \text{RANDOMPICK}(N')$ 
(7)     else
(8)       break
(9)   return  $s$ 
```

Local Search – Stuck in Local Optima

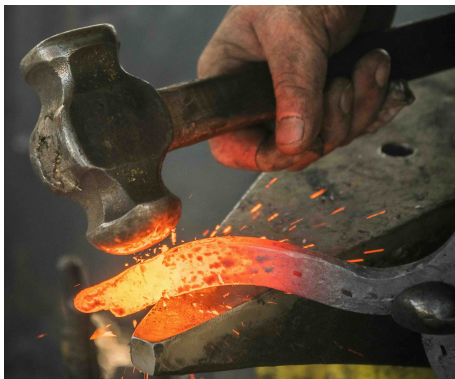


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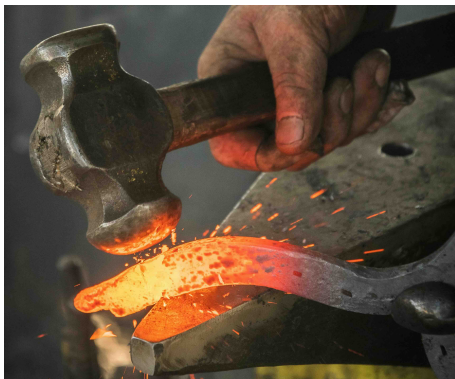
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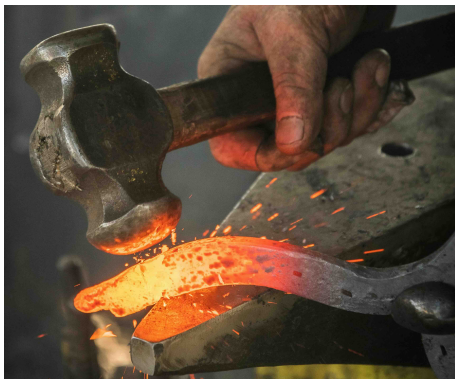
- The local search algorithm may get **stuck in a local optima**.
- Then, how to **escape** from the local optima?
- There are many strategies to **escape** from the local optima.



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- We introduce a **temperature** parameter that controls the **probability** of accepting a **worse solution** for **exploration** purposes.
- The temperature is **gradually decreased** to **reduce** the probability of accepting a worse solution.

SIMULATEDANNEALING()

- (1) $s = s_0$
- (2) $T \leftarrow T_0$
- (3) **for** $k = 0$ **to** n
- (4) $s_{new} \leftarrow \text{GETRANDOMNEIGHBOUR}(s)$
- (5) **if** $P(F(s), F(s_{new}), T) \geq \text{random}(0, 1)$ **then** $s \leftarrow s_{new}$
- (6) $T \leftarrow \text{COOL}(T)$
- (7) **return** s

$P(F(s), F(s_{new}), T)$

- (1) **if** $F(s_{new}) > F(s)$ **then return** 1.0
- (2) **else return** $e^{\frac{F(s_{new}) - F(s)}{T}}$

There are several strategies to **decrease** the temperature (**cooling**):

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- Logarithmic cooling

$$T(t) = \frac{c}{\log(t + d)}$$

- With large c , slow cooling
- Surprisingly, there exists a proof that says that the logarithmic cooling will find the global optimum in infinite time.
- Theoretically interesting, but not practical.

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- Two main ideas:
 - **Memory**: Keep track of **recently visited** solutions and **avoid** them.
 - **Diversification**: Introduce randomness to **explore** the search space.

```

TABUSEARCH()
(1)    $s \leftarrow s_0$ 
(2)    $s_{best} \leftarrow s$ 
(3)    $T \leftarrow []$  // tabu list
(4)   while not stoppingCondition()
(5)        $c_{best} \leftarrow null$ 
(6)       foreach  $c \in \text{GETNEIGHBOURS}(s)$ 
(7)           if  $(c \notin T) \wedge (F(c) > F(c_{best}))$  then  $c_{best} \leftarrow c$ 
(8)        $s \leftarrow c_{best}$ 
(9)       if  $F(c_{best}) > F(s_{best})$  then  $s_{best} \leftarrow c_{best}$ 
(10)      APPEND( $T, c_{best}$ )
(11)      if  $|T| > \text{maxTabuSize}$  then REMOVEAT( $T, 0$ )
(12)  return  $s_{best}$ 

```

Tabu list stores the **recently visited** solutions using a FIFO queue, and we can control the **size** of the tabu list.

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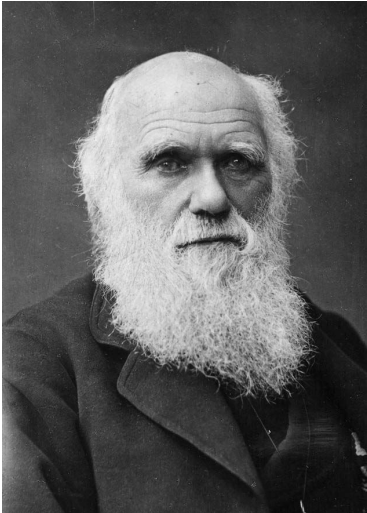
- In common situations, we have a **search budget** (e.g., time, # of fitness evaluations, etc.) for the local search algorithm.
- What if the local search algorithm **stops** but the **budget still remains**?
- We can **restart** the local search algorithm from a **new random solution** to keep searching for the global optimum.

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- Search radius is the **maximum number of moves** required to go **across** the search space.
- For example, consider the **TSP problem** with **20 cities**.
 - **Search Space:** $N! = 20! \approx 2.4 \times 10^{18}$
 - **Search Radius:** $\frac{N(N-1)}{2} = \frac{20 \times 19}{2} = 190$
 - It means that the local search algorithm can find the global optimum within 190 moves in a good situation.

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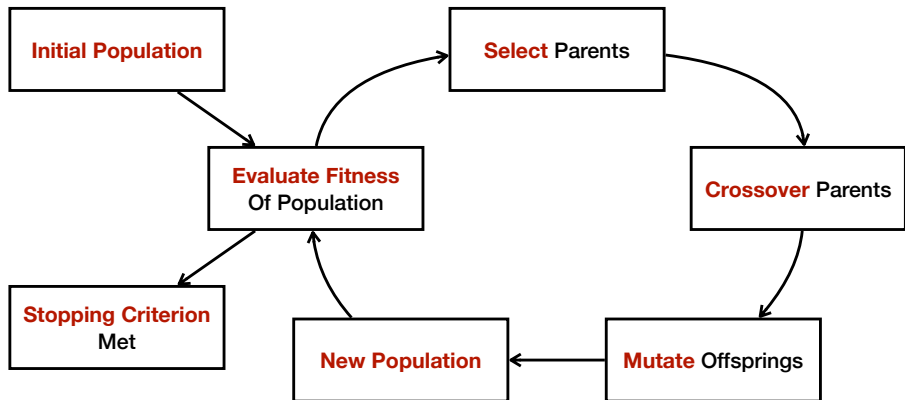


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- In **each generation**, we apply **selection pressure** to **evolve** the population of solutions towards better fitness values.
- Remember: **exploration** and **exploitation**
 - If **too much pressure**, the search converges to a local optimum.
 - If **too little pressure**, the search goes nowhere.



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- This is one of two places where we apply the **selection pressure**.
- The **better** individuals selected as parents, the **more selection pressure** is applied.

Fitness Proportional Selection (FPS): The probability of selecting an individual is proportional to its fitness value.

$$P_{\text{FPS}}(i) = \frac{f(i)}{\sum_{j=1}^{\mu} f(j)}$$

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 $\beta(t) = \min_{i \in P} f(i)$
- **Sigma scaling** – The fitness is transformed by subtracting the mean fitness and dividing by the standard deviation of the fitness values.

$$f'(i) = \max\left(1 + \frac{f(i) - \bar{f}}{2\sigma}, 0.1\right)$$

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There are different ways to utilize ranks to select individuals:

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- **Exponential ranking** – more selection pressure than linear ranking

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Individual	Fitness	Rank	P_{FPS}	$P_{\text{linear}}(s = 1.5)$	$P_{\text{linear}}(s = 2)$	P_{exp}
A	1	0	0.10	0.17	0.00	0.00
B	4	1	0.40	0.33	0.33	0.42
C	5	2	0.50	0.50	0.67	0.58

There are many other selection strategies:

- **Roulette Wheel Selection**
- **Stochastic Universal Sampling (SUS)**
- **Tournament Selection**
- **Over-Selection**
- etc.

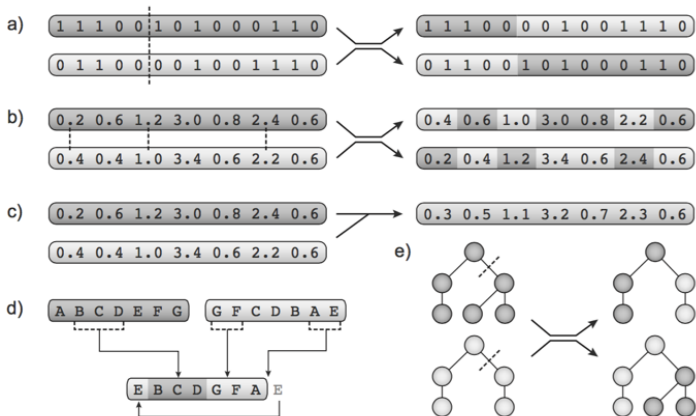


Figure 1.11 Examples of crossover operators. *a)* one-point; *b)* uniform; *c)* arithmetic; *d)* for sequences; *e)* for trees.

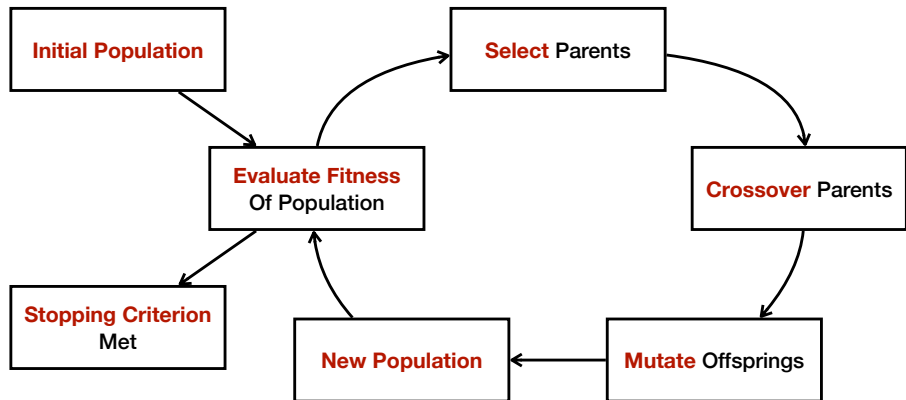
(from “Bio-inspired Artificial Intelligence: Theories, Methods, and Technologies”
by Dario Floreano and Claudio Mattiussi)

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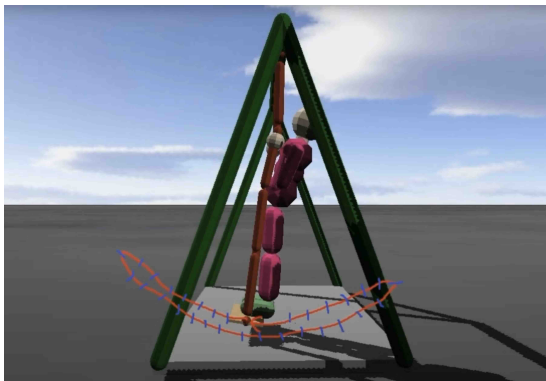
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- The effective way to define the mutation operator is highly **dependent on the problem domain**.



Genetic Algorithms – Example

One interesting example of GA is to **learn** how to **ride a swing**.

https://www.youtube.com/watch?v=Yr_nRnqeDp0



Let's split one cycle of the swing into 32 time steps and define 32-bit representation for the solution (**1** for **standing** and **0** for **sitting**).

- **Knapsack Problem** – NP-hard problem
- **Travelling Salesman Problem (TSP)** – NP-hard problem
- **Program Synthesis** – Automatically generate programs
- **Program Repair** – Automatically repair buggy programs
- **Automotive Design** – Optimize the design of a car
- **Robotics** – Optimize the motion of a robot
- **Molecular structure optimization**
- **Protein folding prediction**
- etc.

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- **Burrs**

- Swiss electrical engineer, George de Mestral, Had to remove **burdock burrs** (seeds) from his cloths and his dog's furs whenever he returned from walks in Alps.
- Eventually, he invented **Velcro hooks** in 1951.

Let's apply the same idea to solve **software engineering problems**.

- Let's mimic the behavior of a **flock of birds!**

Bio-inspired – Particle Swarm Optimization (PSO)

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- **GA** is **competitive** vs. **PSO** is **cooperative**.

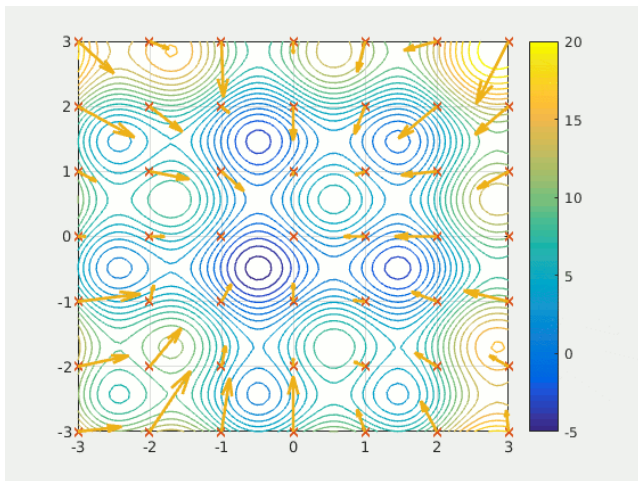
$$x_i^{t+1} = x_i^t + v_i^t$$

$$v_i^{t+1} = \textcircled{1} wv_i^t + \textcircled{2} c_1(p_i - x_i^t) + \textcircled{3} c_2(g - x_i^t)$$

- x_i^t – position of the i -th particle at time t
- v_i^t – velocity of the i -th particle at time t
- p_i – best position of the i -th particle (local best)
- g – best position of the entire flock (global best)

It follows the three rules of the flock of birds.

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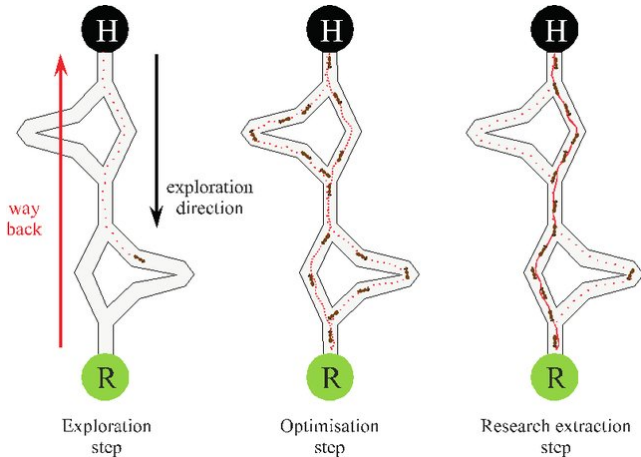


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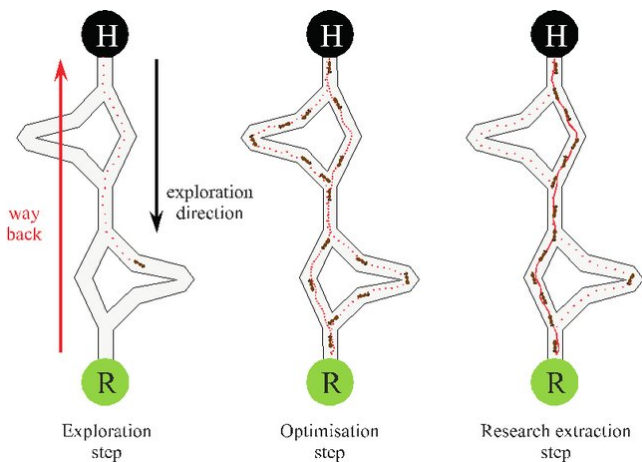
Can we **mimic** the behavior of an **ant colony**?



Ant colony utilizes a **pheromone** to **communicate** with other ants to find the **shortest path** to the food source.



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- 5 By repeating the process, ants converge to the **shortest path**.

- **Probability of ant k choosing edge (i, j) :**

$$p_{i,j}^k = \frac{(\tau_{i,j})^\alpha \cdot (\eta_{i,j})^\beta}{\sum_{h \in J^k} (\tau_{i,h})^\alpha \cdot (\eta_{i,h})^\beta}$$

where $\tau_{i,j}$ is the **amount of pheromone** on edge (i, j) , $\eta_{i,j} = \frac{1}{d_{i,j}}$ is the **inverse of the length** of edge (i, j) , and α and β are the parameters to control the **importance of pheromone and the length** of the edge. J^k is the set of nodes **not yet visited** by ant $1 \leq k \leq m$.

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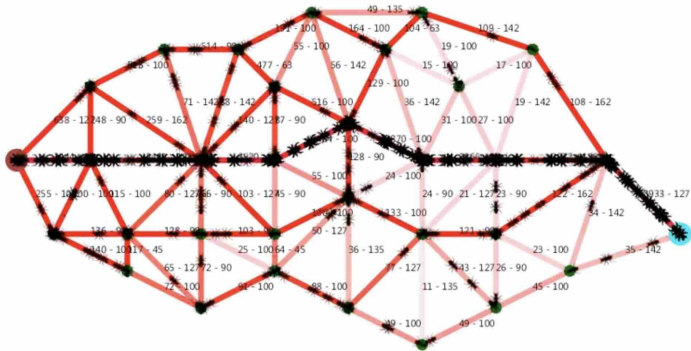
- **Pheromone update:** $\Delta\tau_{i,j} = \frac{Q}{L_k}$, where Q is the constant, and L_k is the length of the tour of ant k .

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- **Evaluation:** $\tau_{i,j} = (1 - \rho)\tau_{i,j} + \sum_{k=1}^m \Delta\tau_{i,j}^k$, where ρ is the **evaporation rate**.



[Link](#)

- When the **graph changes**, the ACO algorithm can **adapt** with the **second-best** solution by **reusing** the pheromone.

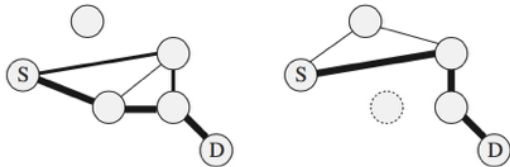


Figure 7.8 *Left:* Virtual ants maintain multiple paths between source and destination nodes. Shorter paths are traversed by more ants (thicker line). *Right:* If a node (or edge) fails, ants immediately use and reinforce the second shortest path available.

Dario Floreano and Claudio Mattiussi, Bio-inspired Artificial Intelligence, MIT Press

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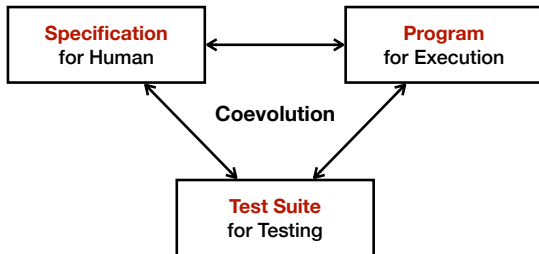
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- **Co-evolutionary Algorithms** – Inspired by the **co-evolution** of species in nature.

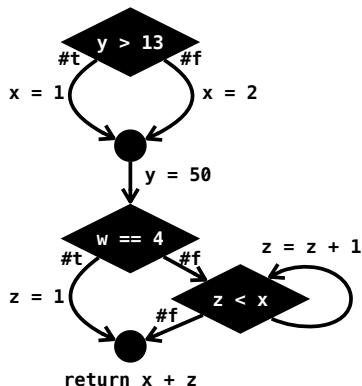


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```

int foo(int x, int y, int w) {
    int z = 0;
    if (y > 13) { x = 1; }
    else { x = 2; }
    y = 50;
    if (w == 4) z = 1;
    else {
        while (z < x) { z = z + 1; }
    }
    return x + z;
}
return x + z;

```

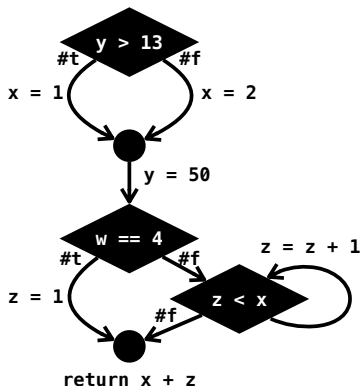


- Our goal is to **automatically generate test cases** to **maximize the coverage** of the software under test.

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- Let's apply the **search-based** approach to **software testing!**

- Convert path conditions into a mathematical **fitness function**.

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- For example, we can define a **fitness function** for branch coverage as:

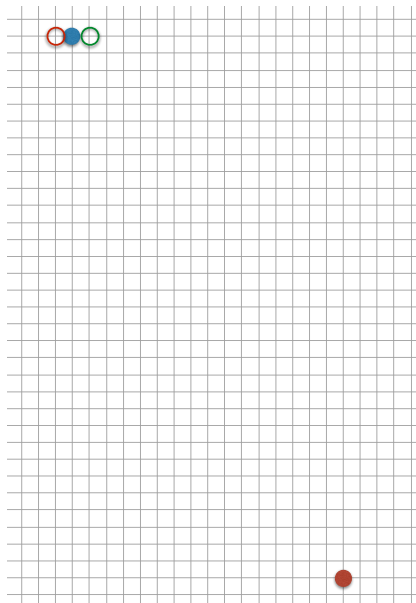
$$[\mathbf{Approach\ Level}] + \text{normalize}([\mathbf{Branch\ Distance}])$$

- **Approach Level** – The number of un-penetrated **nesting levels** surrounding the target branch.
- **Branch Distance** – How close the input came to satisfying the condition of the target branch. For example, if the condition is $x + y == 10$, the branch distance is $|10 - (x + y)|$.

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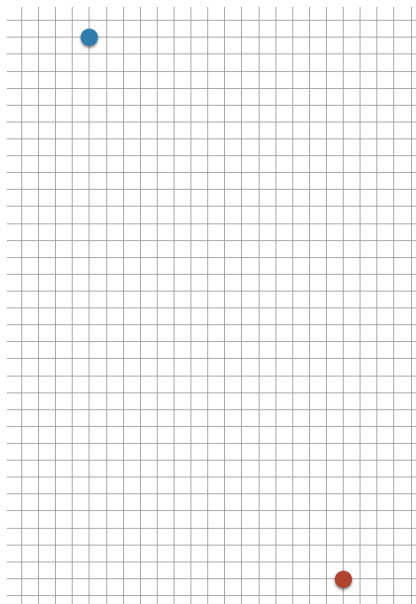
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- Based on the known empirical results, AVM is one of the most effective algorithm for achieving C/C++ structural coverage.
- It has two operation modes:
 - ① **Exploratory Move** – Decide **which direction** results in fitter solutions by exploring neighboring solutions.
 - ② **Pattern Move** – **Accelerate** the search in the selected direction.



Our goal is to **minimize** the fitness function.

(x, y)	Δ	$f(x, y)$	▲/▼
(5, 2)	(-1, 0)	36.23	▲
(7, 2)	(1, 0)	35.34	▼

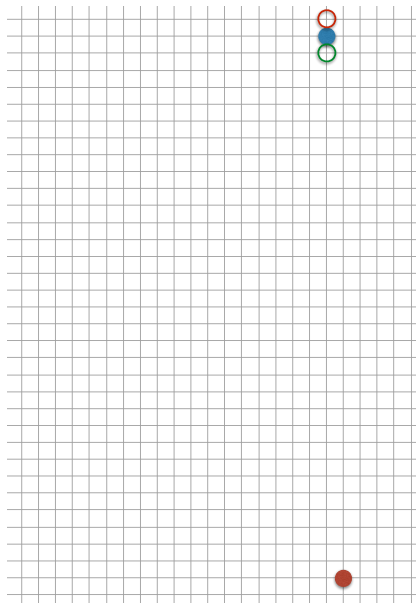
Exploratory Move – $x \uparrow$



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(x, y)	Δ	$f(x, y)$	▲/▼
(7, 2)	(1, 0)	35.34	▼
(9, 2)	(2, 0)	34.53	▼
(13, 2)	(4, 0)	33.24	▼
(21, 2)	(8, 0)	32.01	▼
(37, 2)	(16, 0)	35.34	▲

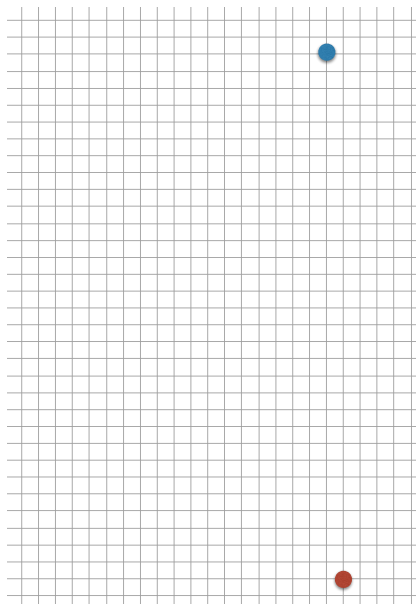
Pattern Move – $x \uparrow$



Our goal is to **minimize** the fitness function.

(x, y)	Δ	$f(x, y)$	▲/▼
(21, 1)	(0, -1)	33.01	▲
(21, 3)	(0, 1)	31.01	▼

Exploratory Move – $y \uparrow$



Our goal is to **minimize** the fitness function.

(x, y)	Δ	$f(x, y)$	▲/▼
(21, 5)	(0, 2)	29.01	▼
(21, 9)	(0, 4)	25.01	▼
(21, 17)	(0, 8)	17.02	▼
(21, 33)	(0, 16)	1.41	▼
(21, 65)	(0, 32)	26.03	▲

Pattern Move – $y \uparrow$

Our goal is to **minimize** the fitness function.

After one or two more iterations, we can find the **optimal solution**.

$$(x, y) = (22, 34)$$

Summary

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- Dynamic Symbolic Execution (DSE)

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