

Lecture 6 – Dynamic Symbolic Execution (DSE)

AAA705: Software Testing and Quality Assurance

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2024 Spring

- **Search Based Software Engineering (SBSE)**

- Fitness Landscape
- Local Search
 - Hill Climbing
 - Simulated Annealing
 - Tabu Search
- Genetic Algorithms
 - Selection Strategies
 - Crossover Operators
 - Mutation Operators
- Bio-inspired Algorithms
 - Particle Swarm Optimization (PSO)
 - Ant Colony Optimization (ACO)

- **Search Based Software Testing (SBST)**

- Alternating Variable Method (AVM)

Sometimes called **structural testing** because it uses the **internal structure** of the program to derive test cases.

- **Coverage Criteria**

- The adequacy of a test suite is measured in terms of the **coverage** of the program's internal structure.

- **Search Based Software Testing (SBST)**

- A technique that uses **meta-heuristic search** algorithms to maximize/minimize a certain **fitness function**.

- **Dynamic Symbolic Execution (DSE)**

- A technique that systematically explores the input space using **symbolic execution** with **dynamic analysis**.

Let's focus on the **Dynamic Symbolic Execution (DSE)** technique.

1. Symbolic Execution

- Basic Idea

- Satisfiability Modulo Theories (SMT)

- Limitations of Symbolic Execution

2. Dynamic Symbolic Execution (DSE)

- Search Heuristics

- Example – Hash Function

- Example – Loops

- Example – Data Structures

- Realistic Implementation

- Other Hybrid Analysis Techniques

1. Symbolic Execution

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- Other Hybrid Analysis Techniques

- Random testing has a **limitation** that it sometimes fails or takes a long time to find bugs if they can only be triggered **under specific conditions**.
- For example, consider the following program.

```
void testme (int x) {  
    if (x == 93589) {  
        ERROR  
    }  
}
```

- The bug can only be triggered when the input is $x = 93589$.
- It means that the probability of triggering the bug is as follows when the integer input is randomly generated:

$$\frac{1}{2^{32}} \approx 0.00000000023283\%$$

- **Symbolic execution** is a program analysis technique that explores the paths of a program with **symbolic values** as inputs and collects **constraints** on the inputs.
- **1976** – A system to generate test data and symbolically execute programs (Lori Clarke).
- **1977** – Symbolic execution and program testing (James King).
- **2005 to present** – Practical symbolic execution
 - Using advanced constraint solvers (SMT solvers)
 - Heuristics to control exponential path explosion
 - Heap modeling and reasoning about complex data structures
 - Environment modeling
 - Dealing with solver limitations

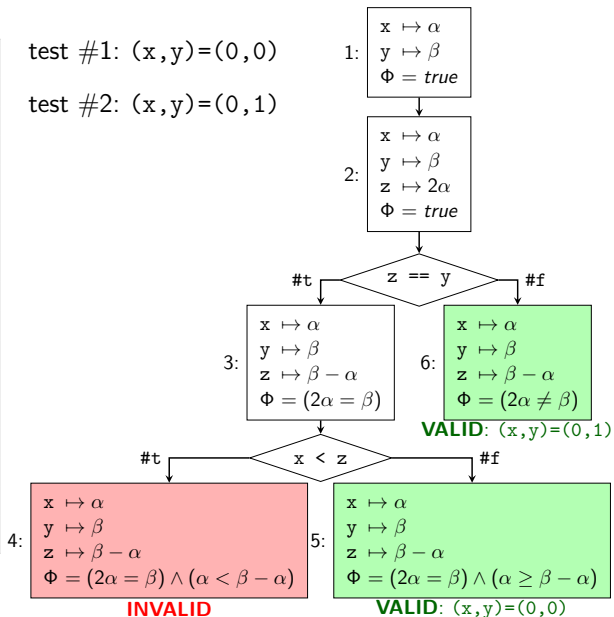
- 1 Execute the program on **symbolic values** Ω .
- 2 A **symbolic state** $\sigma \in \mathbb{S} = \mathbb{X} \xrightarrow{\text{fin}} \Omega$ is a finite mapping from variables \mathbb{X} to symbolic values Ω .
- 3 A **path condition** $\Phi = \phi_1 \wedge \dots \wedge \phi_n$ is a quantifier-free formula over the symbolic inputs that encodes all branch decisions taken so far.
- 4 All paths in the program form its **execution tree**, in which some paths are **feasible** and some are **infeasible**.

Symbolic Execution – Basic Idea

```
void f(int x, int y) {  
    /* 1 */  
    int z = 2 * x;  
    /* 2 */  
    if (z == y) {  
        z = y - x;  
        /* 3 */  
        if (x < z) {  
            /* 4 */  
            ERROR;  
        } else {  
            /* 5 */  
        }  
    } else {  
        /* 6 */  
    }  
}
```

test #1: $(x,y)=(0,0)$

test #2: $(x,y)=(0,1)$



- Then, how to check the **satisfiability** of a path condition Φ ?
- Most symbolic execution tools use **Satisfiability Modulo Theories (SMT)** solvers (e.g., Z3, CVC4) to check it.
- An SMT solver takes a **first-order logic formula** and returns whether it is **satisfiable** or not using various background theories, such as arithmetic, arrays, bit-vectors, algebraic data types, etc.

- Check the satisfiability of the following formula using an SMT solver.

$$b + 2 = c \wedge f(\text{read}(\text{write}(a, b, 3), c - 2)) \neq f(c - b + 1)$$

- **Substitute** c by $b + 2$ in the second part of the formula.

$$b + 2 = c \wedge f(\text{read}(\text{write}(a, b, b + 2 - 2), b)) \neq f(b + 2 - b + 1)$$

- **Arithmetic simplification** of the formula.

$$b + 2 = c \wedge f(\text{read}(\text{write}(a, b, 3), b)) \neq f(3)$$

- **Applying array theory axiom** – $\forall a, i, v. \text{read}(\text{write}(a, i, v), i) = v.$

$$b + 2 = c \wedge f(3) \neq f(3) \quad (\text{UNSAT})$$

Z3¹ is one of the most popular SMT solvers developed by Microsoft Research and used in many symbolic execution tools.

For example, the following Z3 script returns `unsat`.

```
(declare-const a (Array Int Int))
(declare-const b Int)
(declare-const c Int)
(declare-fun f (Int) Int)
(assert (= (+ b 2) c))
(assert (not (= (f (select (store a b 3) (- c 2))) (f (+ (- c b) 1)))))
(check-sat) ; => unsat
```

Or, the following script returns a satisfying assignment $x = 0$ and $y = 0$.

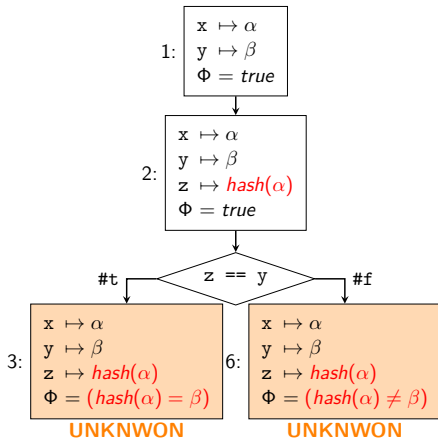
```
(declare-const x Int)
(declare-const y Int)
(assert (and (= (* 2 x) y) (>= x (- y x))))
(check-sat) ; => sat
(get-model) ; => (x, y) = (0, 0)
```

¹<https://github.com/Z3Prover/z3>

We cannot solve path condition Φ containing the hash function using SMT solver.

```

void f(int x, int y) {
    /* 1 */
    int z = hash(x);
    /* 2 */
    if (z == y) {
        z = y - x;
        /* 3 */
        if (x < z) {
            /* 4 */
            ERROR;
        } else {
            /* 5 */
        }
    } else {
        /* 6 */
    }
}
    
```



1. Symbolic Execution

Basic Idea

Satisfiability Modulo Theories (SMT)

Limitations of Symbolic Execution

2. Dynamic Symbolic Execution (DSE)

Search Heuristics

Example – Hash Function

Example – Loops

Example – Data Structures

Realistic Implementation

Other Hybrid Analysis Techniques

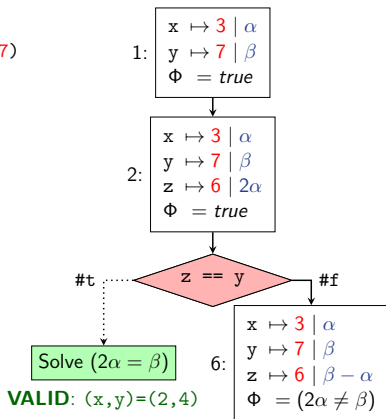
- **Dynamic Symbolic Execution (DSE)** is a technique that combines **concrete execution** with **symbolic execution** to overcome the limitations of symbolic execution.
- It is sometimes called **concolic testing** because it combines both **concrete** and **symbolic** execution to generate **test cases**.
- It stores both the **concrete** values and the **symbolic** values during the execution of the program, and solves the path condition to **guide execution** at branch points.
- The concrete values are used to **simplify** the path condition.

```

void f(int x, int y) {
    /* 1 */
    int z = 2 * x;
    /* 2 */
    if (z == y) {
        z = y - x;
        /* 3 */
        if (x < z) {
            /* 4 */
            ERROR;
        } else {
            /* 5 */
        }
    } else {
        /* 6 */
    }
}

```

test #1: (x,y)=(3,7)

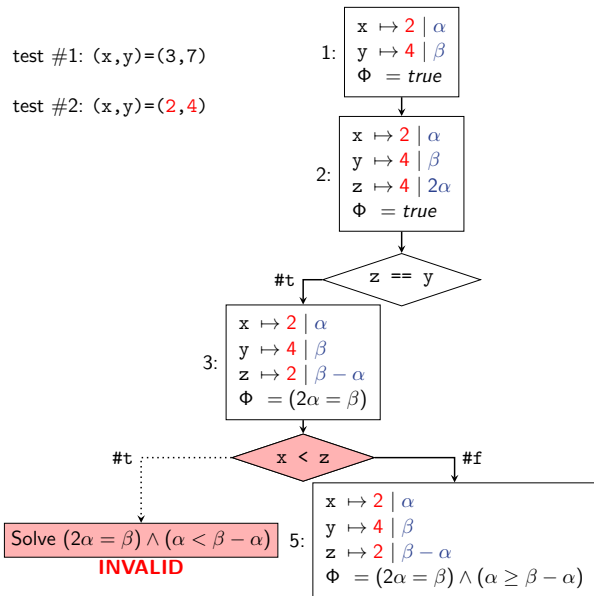


```

void f(int x, int y) {
  /* 1 */
  int z = 2 * x;
  /* 2 */
  if (z == y) {
    z = y - x;
    /* 3 */
    if (x < z) {
      /* 4 */
      ERROR;
    } else {
      /* 5 */
    }
  } else {
    /* 6 */
  }
}
    
```

test #1: (x,y)=(3,7)

test #2: (x,y)=(2,4)



Algorithm 1. Concolic Testing

Input : Program P , budget N , initial input v_0

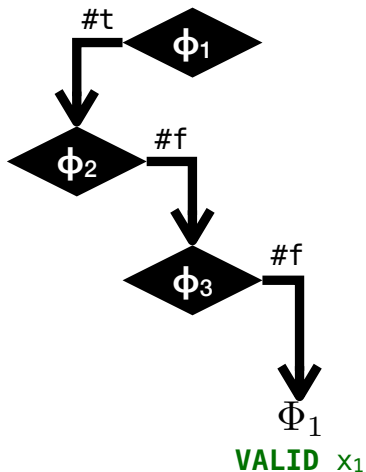
Output : The number of branches covered

- 1: $T \leftarrow \langle \rangle$
 - 2: $v \leftarrow v_0$
 - 3: **for** $m = 1$ to N **do**
 - 4: $\Phi_m \leftarrow \text{RunProgram}(P, v)$
 - 5: $T \leftarrow T \cdot \Phi_m$
 - 6: **repeat**
 - 7: $(\Phi, \phi_i) \leftarrow \text{Choose}(T)$ $(\Phi = \phi_1 \wedge \dots \wedge \phi_n)$
 - 8: **until** $\text{SAT}(\bigwedge_{j < i} \phi_j \wedge \neg \phi_i)$
 - 9: $v \leftarrow \text{model}(\bigwedge_{j < i} \phi_j \wedge \neg \phi_i)$
 - 10: **return** $|\text{Branches}(T)|$
-

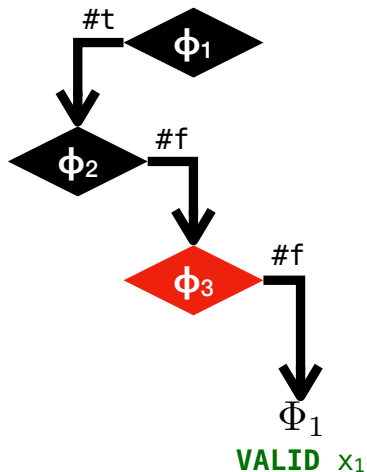
In each iteration, DSE **chooses** a path to explore based on the path condition Φ using a specific **search heuristic**.

- **Random Search** – Randomly selects a branch from the most recently visited execution path.
- **Control-Flow Directed Search (CFDS)** – Selects the uncovered branch **closest** to the last branch in the current execution path.
- **Context-Guided Search (CGS)** – Performs a **breadth-first search (BFS)** on the execution tree by excluding branches whose **contexts** (i.e., the last d preceding branches) are already explored.
- **Depth-First Search (DFS)**
- etc.

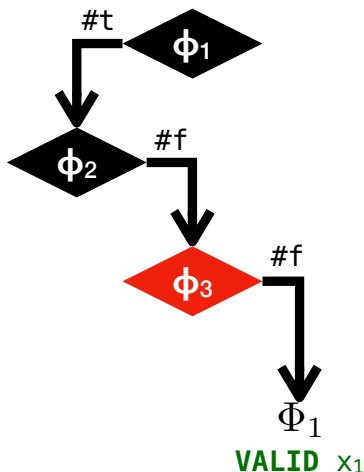
This is the **control-flow directed search (CFDS)** heuristic.



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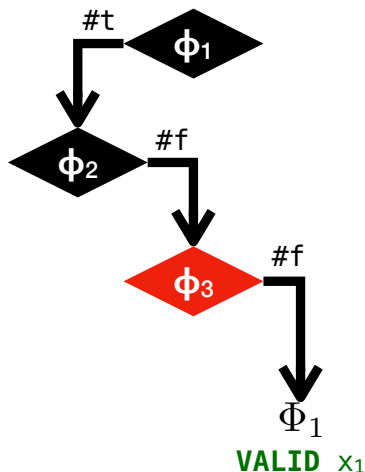


This is the **control-flow directed search (CFDS)** heuristic.



$$\text{Solve } \phi_1 \wedge \neg\phi_2 \wedge \phi_3$$

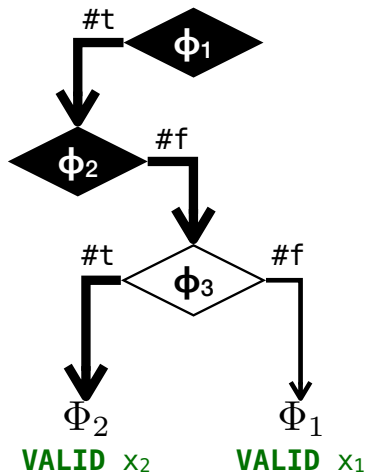
This is the **control-flow directed search (CFDS)** heuristic.



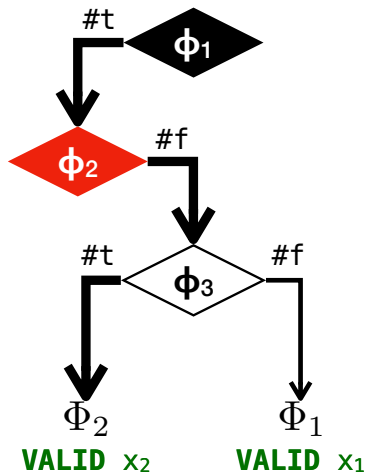
Solve $\phi_1 \wedge \neg\phi_2 \wedge \phi_3$

VALID x_2

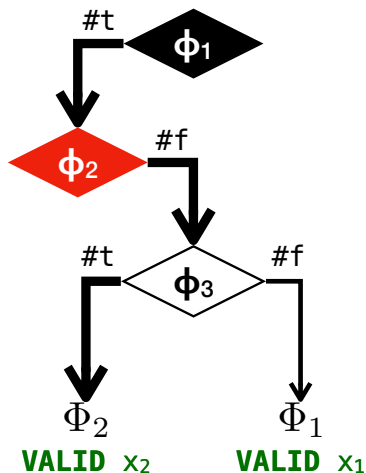
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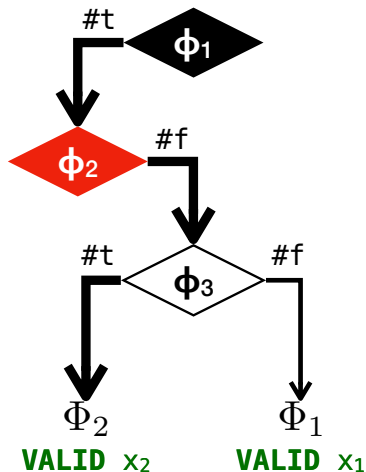


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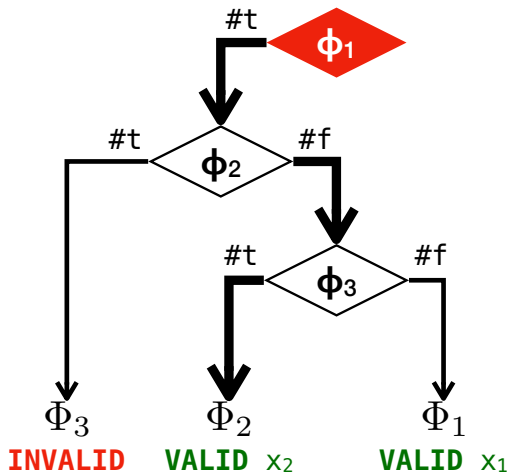
Solve $\phi_1 \wedge \phi_2$

This is the **control-flow directed search (CFDS)** heuristic.



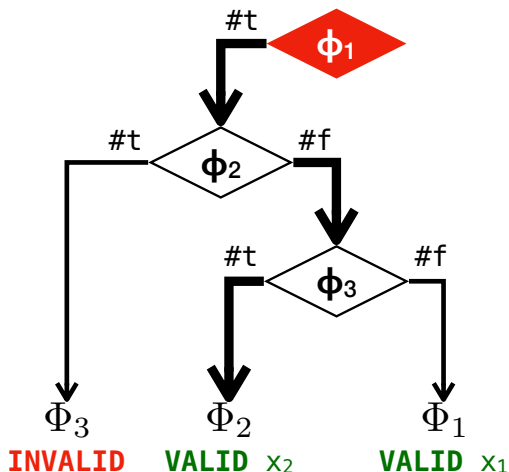
Solve $\phi_1 \wedge \phi_2$
INVALID

This is the **control-flow directed search (CFDS)** heuristic.

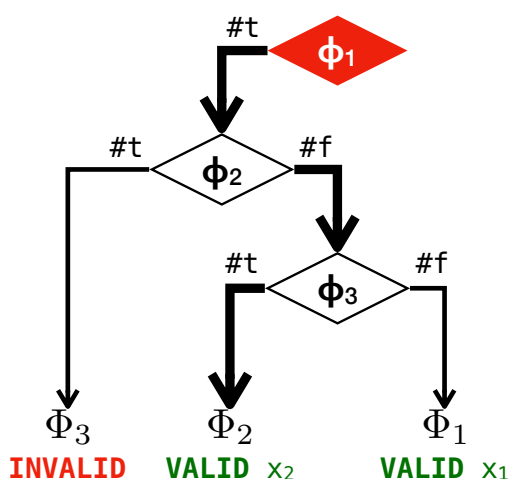


This is the **control-flow directed search (CFDS)** heuristic.

Solve $\neg\phi_1$



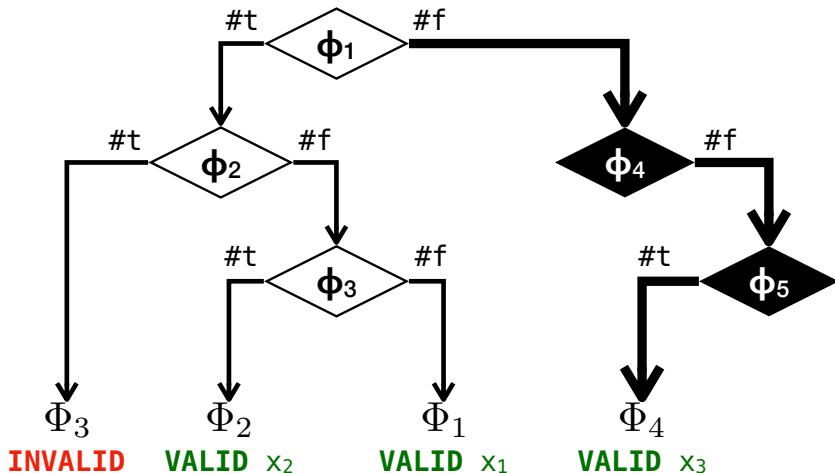
This is the **control-flow directed search (CFDS)** heuristic.



Solve $\neg\phi_1$

VALID x_3

This is the **control-flow directed search (CFDS)** heuristic.



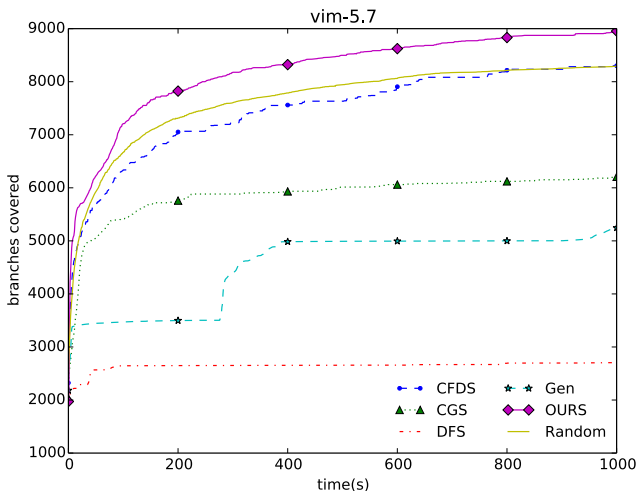
[ICSE'18] S. Cha et al., “Automatically Generating Search Heuristics for Concolic Testing”

$$\text{Choose}_\theta(\langle \Phi_1, \dots, \Phi_m \rangle) = (\Phi_m, \underset{\phi_j \in \Phi_m}{\operatorname{argmax}} \operatorname{score}_\theta(\phi_j))$$

Followings are 12 static and 28 dynamic branch features used for learning.

| # | Description |
|----|--|
| 1 | branch in the main function |
| 2 | true branch of a loop |
| 3 | false branch of a loop |
| 4 | nested branch |
| 5 | branch containing external function calls |
| 6 | branch containing integer expressions |
| 7 | branch containing constant strings |
| 8 | branch containing pointer expressions |
| 9 | branch containing local variables |
| 10 | branch inside a loop body |
| 11 | true branch of a case statement |
| 12 | false branch of a case statement |
| 13 | first 10% branches of a path |
| 14 | last 10% branches of a path |
| 15 | branch appearing most frequently in a path |

[ICSE'18] S. Cha et al., "Automatically Generating Search Heuristics for Concolic Testing"



Example – Hash Function

```
void f(int x, int y) {  
    *  
    int z = hash(x);  
  
    if (z == y) {  
        z = y - x;  
  
        if (x < z) {  
  
            ERROR;  
        } else {  
  
        }  
    } else {  
  
    }  
}
```

| \mathbb{X} | \mathbb{V} | Ω |
|--------------|--------------|----------|
| x | 3 | α |
| y | 7 | β |
| z | | |
| Φ | true | |

```

void f(int x, int y) {

    int z = hash(x);
    *
    if (z == y) {
        z = y - x;

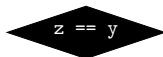
        if (x < z) {

            ERROR;
        } else {

        }
    } else {

    }
}
    
```

| \mathbb{X} | \mathbb{V} | Ω |
|--------------|--------------|----------------|
| x | 3 | α |
| y | 7 | β |
| z | 182039482 | $hash(\alpha)$ |
| Φ | true | |



```

void f(int x, int y) {

    int z = hash(x);

    if (z == y) {
        z = y - x;

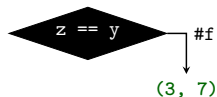
        if (x < z) {

            ERROR;
        } else {

            *
        }
    } else {

    }
}
    
```

| \mathbb{X} | \mathbb{V} | Ω |
|--------------|-----------------------------|----------------|
| x | 3 | α |
| y | 7 | β |
| z | 182039482 | $hash(\alpha)$ |
| Φ | $(hash(\alpha) \neq \beta)$ | |




```

void f(int x, int y) {

    int z = hash(x);

    if (z == y) {
        z = y - x;

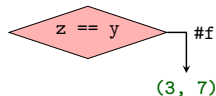
        if (x < z) {

            ERROR;
        } else {

            *
        }
    } else {

    }
}
    
```

| \mathbb{X} | \mathbb{V} | Ω |
|--------------|-----------------------------|----------------|
| x | 3 | α |
| y | 7 | β |
| z | 182039482 | $hash(\alpha)$ |
| Φ | $(hash(\alpha) \neq \beta)$ | |



We can utilize the current concrete values.

$(hash(\alpha) = \beta)$ is **SAT**
 when $(x, y) = (3, 182039482)$.

```

void f(int x, int y) {
    *
    int z = hash(x);

    if (z == y) {
        z = y - x;

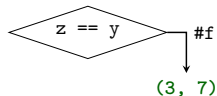
        if (x < z) {

            ERROR;
        } else {

        }
    } else {

    }
}
    
```

| \mathbb{X} | \mathbb{V} | Ω |
|--------------|--------------|----------|
| x | 3 | α |
| y | 182039482 | β |
| z | | |
| Φ | true | |



```

void f(int x, int y) {

    int z = hash(x);
    *
    if (z == y) {
        z = y - x;

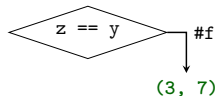
        if (x < z) {

            ERROR;
        } else {

        }
    } else {

    }
}
    
```

| \mathbb{X} | \mathbb{V} | Ω |
|--------------|--------------|----------------|
| x | 3 | α |
| y | 182039482 | β |
| z | 182039482 | $hash(\alpha)$ |
| Φ | true | |



```

void f(int x, int y) {

    int z = hash(x);

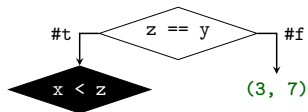
    if (z == y) {
        z = y - x;
        *
        if (x < z) {

            ERROR;
        } else {

        }
    } else {

    }
}
    
```

| \mathbb{X} | \mathbb{V} | Ω |
|--------------|--------------------------|------------------|
| x | 3 | α |
| y | 182039482 | β |
| z | 182039479 | $\beta - \alpha$ |
| Φ | $(hash(\alpha) = \beta)$ | |



```

void f(int x, int y) {

    int z = hash(x);

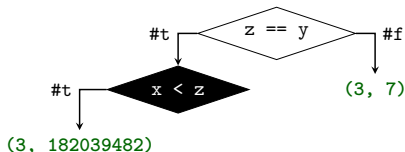
    if (z == y) {
        z = y - x;

        if (x < z) {
            *
            ERROR;
        } else {

        }
    } else {

    }
}
    
```

| \mathbb{X} | \mathbb{V} | Ω |
|--------------|---|------------------|
| x | 3 | α |
| y | 182039482 | β |
| z | 182039479 | $\beta - \alpha$ |
| Φ | $(hash(\alpha) = \beta) \wedge (\alpha < \beta - \alpha)$ | |



We found an error!

```

void f(int x, int y) {

    int z = hash(x);

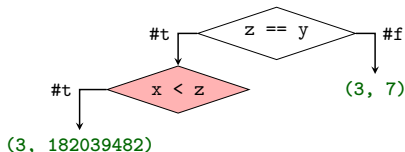
    if (z == y) {
        z = y - x;

        if (x < z) {
            *
            ERROR;
        } else {

        }
    } else {

    }
}
    
```

| \mathbb{X} | \mathbb{V} | Ω |
|--------------|---|------------------|
| x | 3 | α |
| y | 182039482 | β |
| z | 182039479 | $\beta - \alpha$ |
| Φ | $(hash(\alpha) = \beta) \wedge (\alpha < \beta - \alpha)$ | |



Unfortunately,
 $(hash(\alpha) = \beta) \wedge (\alpha \geq \beta - \alpha)$ is
UNKNOWN.

Example – Loops

```
void f(int x) {  
    *  
    int arr[] = {3, 7, 2};  
    int i = 0;  
  
    while (i < 3) {  
        if (arr[i] == x) break;  
        i++;  
    }  
  
    return i;  
}
```

| | | |
|--------------|--------------|----------|
| \mathbb{X} | \mathbb{V} | Ω |
| x | 0 | α |
| arr | | |
| i | | |
| Φ | <i>true</i> | |

```
void f(int x) {  
  
    int arr[] = {3, 7, 2};  
    int i = 0;  
    *  
    while (i < 3) {  
  
        if (arr[i] == x) break;  
        i++;  
  
    }  
  
    return i;  
}
```

| | | |
|--------------|--------------|----------|
| \mathbb{X} | \mathbb{V} | Ω |
| x | 0 | α |
| arr | {3, 7, 2} | |
| i | 0 | |
| Φ | <i>true</i> | |

Example – Loops

```
void f(int x) {  
  
    int arr[] = {3, 7, 2};  
    int i = 0;  
  
    while (i < 3) {  
        *  
        if (arr[i] == x) break;  
        i++;  
    }  
  
    return i;  
}
```

| \mathbb{X} | \mathbb{V} | Ω |
|--------------|--------------|----------|
| x | 0 | α |
| arr | {3, 7, 2} | |
| i | 0 | |
| Φ | <i>true</i> | |

```

void f(int x) {

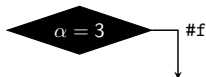
    int arr[] = {3, 7, 2};
    int i = 0;

    while (i < 3) {

        if (arr[i] == x) break;
        i++;
        *
    }

    return i;
}
    
```

| | | |
|--------------|-------------------|----------|
| \mathbb{X} | \mathbb{V} | Ω |
| x | 0 | α |
| arr | {3, 7, 2} | |
| i | 1 | |
| Φ | $(\alpha \neq 3)$ | |



```

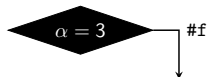
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    int arr[] = {3, 7, 2};
    int i = 0;

    while (i < 3) {
        *
        if (arr[i] == x) break;
        i++;
    }

    return i;
}
    
```

| | | |
|--------------|-------------------|----------|
| \mathbb{X} | \mathbb{V} | Ω |
| x | 0 | α |
| arr | {3, 7, 2} | |
| i | 1 | |
| Φ | $(\alpha \neq 3)$ | |



```

void f(int x) {

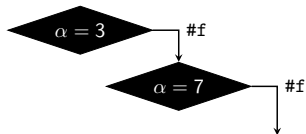
    int arr[] = {3, 7, 2};
    int i = 0;

    while (i < 3) {

        if (arr[i] == x) break;
        i++;
        *
    }

    return i;
}
    
```

| \mathbb{X} | \mathbb{V} | Ω |
|--------------|--|----------|
| x | 0 | α |
| arr | {3, 7, 2} | |
| i | 2 | |
| Φ | $(\alpha \neq 3) \wedge (\alpha \neq 7)$ | |



```

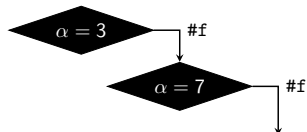
void f(int x) {

    int arr[] = {3, 7, 2};
    int i = 0;

    while (i < 3) {
        *
        if (arr[i] == x) break;
        i++;
    }

    return i;
}
    
```

| | | |
|--------------|--|----------|
| \mathbb{X} | \mathbb{V} | Ω |
| x | 0 | α |
| arr | {3, 7, 2} | |
| i | 2 | |
| Φ | $(\alpha \neq 3) \wedge (\alpha \neq 7)$ | |



```

void f(int x) {

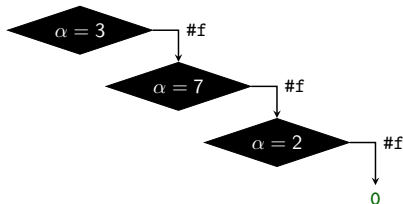
    int arr[] = {3, 7, 2};
    int i = 0;

    while (i < 3) {

        if (arr[i] == x) break;
        i++;
        *
    }

    return i;
}
    
```

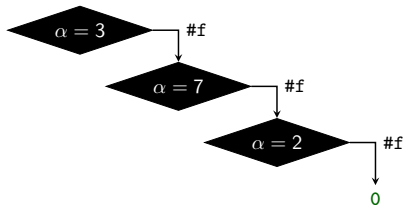
| \mathbb{X} | \mathbb{V} | Ω |
|--------------|---|----------|
| x | 0 | α |
| arr | {3, 7, 2} | |
| i | 3 | |
| Φ | $(\alpha \neq 3) \wedge (\alpha \neq 7) \wedge (\alpha \neq 2)$ | |



Example – Loops

```
void f(int x) {  
  
    int arr[] = {3, 7, 2};  
    int i = 0;  
  
    while (i < 3) {  
  
        if (arr[i] == x) break;  
        i++;  
  
    }  
    *  
    return i;  
}
```

| \mathbb{X} | \mathbb{V} | Ω |
|--------------|---|----------|
| x | 0 | α |
| arr | {3, 7, 2} | |
| i | 3 | |
| Φ | $(\alpha \neq 3) \wedge (\alpha \neq 7) \wedge (\alpha \neq 2)$ | |



```

void f(int x) {

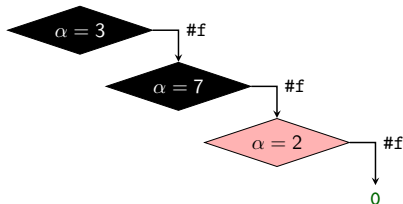
    int arr[] = {3, 7, 2};
    int i = 0;

    while (i < 3) {

        if (arr[i] == x) break;
        i++;

    }
    *
    return i;
}
    
```

| \mathbb{X} | \mathbb{V} | Ω |
|--------------|---|----------|
| x | 0 | α |
| arr | {3, 7, 2} | |
| i | 3 | |
| Φ | $(\alpha \neq 3) \wedge (\alpha \neq 7) \wedge (\alpha \neq 2)$ | |

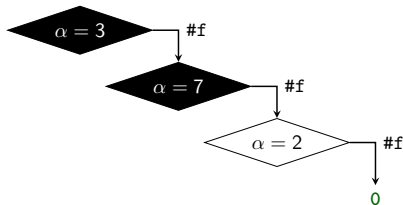


$(\alpha \neq 3) \wedge (\alpha \neq 7) \wedge (\alpha = 2)$ is **SAT**
when $x = 2$

Example – Loops

```
void f(int x) {  
    *  
    int arr[] = {3, 7, 2};  
    int i = 0;  
  
    while (i < 3) {  
        if (arr[i] == x) break;  
        i++;  
    }  
  
    return i;  
}
```

| \mathbb{X} | \mathbb{V} | Ω |
|--------------|--------------|----------|
| x | 2 | α |
| arr | | |
| i | | |
| Φ | <i>true</i> | |



```

void f(int x) {

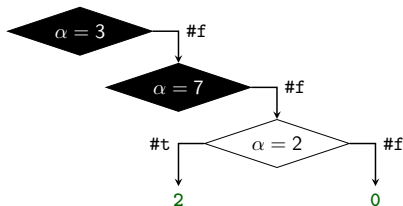
    int arr[] = {3, 7, 2};
    int i = 0;

    while (i < 3) {

        if (arr[i] == x) break;
        i++;

    }
    *
    return i;
}
    
```

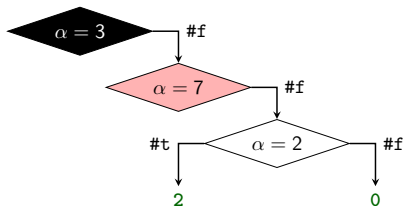
| \mathbb{X} | \mathbb{V} | Ω |
|--------------|--|----------|
| x | 2 | α |
| arr | {3, 7, 2} | |
| i | 2 | |
| Φ | $(\alpha \neq 3) \wedge (\alpha \neq 7) \wedge (\alpha = 2)$ | |



Example – Loops

```
void f(int x) {  
  
    int arr[] = {3, 7, 2};  
    int i = 0;  
  
    while (i < 3) {  
  
        if (arr[i] == x) break;  
        i++;  
  
    }  
    *  
    return i;  
}
```

| \mathbb{X} | \mathbb{V} | Ω |
|--------------|--|----------|
| x | 2 | α |
| arr | {3, 7, 2} | |
| i | 2 | |
| Φ | $(\alpha \neq 3) \wedge (\alpha \neq 7) \wedge (\alpha = 2)$ | |

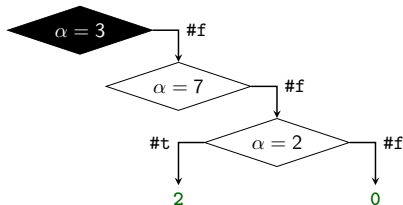


$(\alpha \neq 3) \wedge (\alpha = 7)$ is **SAT**
when $x = 7$

Example – Loops

```
void f(int x) {  
    *  
    int arr[] = {3, 7, 2};  
    int i = 0;  
  
    while (i < 3) {  
        if (arr[i] == x) break;  
        i++;  
    }  
  
    return i;  
}
```

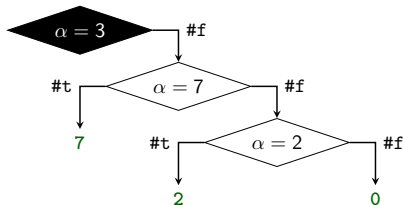
| \mathbb{X} | \mathbb{V} | Ω |
|--------------|--------------|----------|
| x | 7 | α |
| arr | | |
| i | | |
| Φ | true | |



Example – Loops

```
void f(int x) {  
  
    int arr[] = {3, 7, 2};  
    int i = 0;  
  
    while (i < 3) {  
  
        if (arr[i] == x) break;  
        i++;  
  
    }  
    *  
    return i;  
}
```

| \mathbb{X} | \mathbb{V} | Ω |
|--------------|---------------------------------------|----------|
| x | 7 | α |
| arr | {3, 7, 2} | |
| i | 1 | |
| Φ | $(\alpha \neq 3) \wedge (\alpha = 7)$ | |



```

void f(int x) {

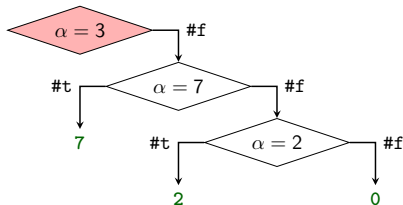
    int arr[] = {3, 7, 2};
    int i = 0;

    while (i < 3) {

        if (arr[i] == x) break;
        i++;

    }
    *
    return i;
}
    
```

| \mathbb{X} | \mathbb{V} | Ω |
|--------------|---------------------------------------|----------|
| x | 7 | α |
| arr | {3, 7, 2} | |
| i | 1 | |
| Φ | $(\alpha \neq 3) \wedge (\alpha = 7)$ | |

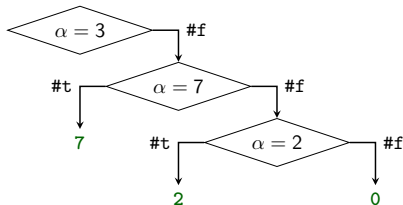


$(\alpha = 3)$ is **SAT**
when $x = 3$

Example – Loops

```
void f(int x) {  
    *  
    int arr[] = {3, 7, 2};  
    int i = 0;  
  
    while (i < 3) {  
        if (arr[i] == x) break;  
        i++;  
    }  
  
    return i;  
}
```

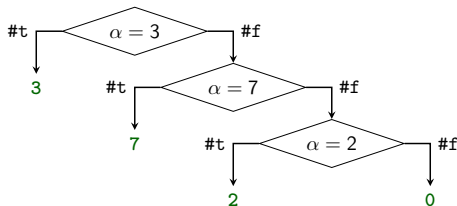
| \mathbb{X} | \mathbb{V} | Ω |
|--------------|--------------|----------|
| x | 3 | α |
| arr | | |
| i | | |
| Φ | <i>true</i> | |



Example – Loops

```
void f(int x) {  
  
    int arr[] = {3, 7, 2};  
    int i = 0;  
  
    while (i < 3) {  
  
        if (arr[i] == x) break;  
        i++;  
  
    }  
    *  
    return i;  
}
```

| \mathbb{X} | \mathbb{V} | Ω |
|--------------|----------------|----------|
| x | 3 | α |
| arr | {3, 7, 2} | |
| i | 0 | |
| Φ | $(\alpha = 3)$ | |




```

class Node {
    int data;
    Node* next;
};

void f(int x, Node *p) {
    *
    if (x > 0)

        if (p != NULL)

            if (x*2+1 == p->data)

                if (p->next == p)

                    ERROR;

    return 0;
}
    
```

| | | |
|--------------|--------------|----------|
| \mathbb{X} | \mathbb{V} | Ω |
| x | 0 | α |
| p | NULL | β |
| Φ | true | |

```

class Node {
    int data;
    Node* next;
};

void f(int x, Node *p) {

    if (x > 0)

        if (p != NULL)

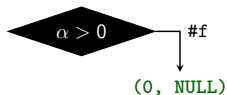
            if (x*2+1 == p->data)

                if (p->next == p)

                    ERROR;

                *
            return 0;
    }
    
```

| | | |
|--------------|-------------------|----------|
| \mathbb{X} | \mathbb{V} | Ω |
| x | 0 | α |
| p | NULL | β |
| Φ | $(\alpha \leq 0)$ | |



```

class Node {
    int data;
    Node* next;
};

void f(int x, Node *p) {

    if (x > 0)

        if (p != NULL)

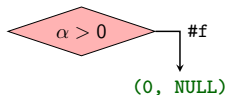
            if (x*2+1 == p->data)

                if (p->next == p)

                    ERROR;

                *
            return 0;
    }
    
```

| | | |
|--------------|-------------------|----------|
| \mathbb{X} | \mathbb{V} | Ω |
| x | 0 | α |
| p | NULL | β |
| Φ | $(\alpha \leq 0)$ | |



$(\alpha > 0)$ is **SAT**
 when $(x, p) = (42, \text{NULL})$

```

class Node {
    int data;
    Node* next;
};

void f(int x, Node *p) {
    *
    if (x > 0)

        if (p != NULL)

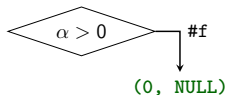
            if (x*2+1 == p->data)

                if (p->next == p)

                    ERROR;

    return 0;
}
    
```

| | | |
|--------------|--------------|----------|
| \mathbb{X} | \mathbb{V} | Ω |
| x | 42 | α |
| p | NULL | β |
| Φ | true | |



```

class Node {
    int data;
    Node* next;
};

void f(int x, Node *p) {

    if (x > 0)
        *
        if (p != NULL)

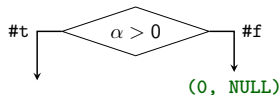
            if (x*2+1 == p->data)

                if (p->next == p)

                    ERROR;

    return 0;
}
    
```

| | | |
|--------------|----------------|----------|
| \mathbb{X} | \mathbb{V} | Ω |
| x | 42 | α |
| p | NULL | β |
| Φ | $(\alpha > 0)$ | |



```

class Node {
    int data;
    Node* next;
};

void f(int x, Node *p) {

    if (x > 0)

        if (p != NULL)

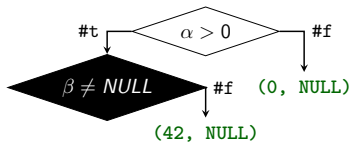
            if (x*2+1 == p->data)

                if (p->next == p)

                    ERROR;

                *
            return 0;
    }
    
```

| | | |
|--------------|---|----------|
| \mathbb{X} | \mathbb{V} | Ω |
| x | 42 | α |
| p | NULL | β |
| Φ | $(\alpha > 0) \wedge (\beta = \text{NULL})$ | |



```

class Node {
    int data;
    Node* next;
};

void f(int x, Node *p) {

    if (x > 0)

        if (p != NULL)

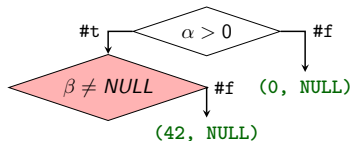
            if (x*2+1 == p->data)

                if (p->next == p)

                    ERROR;

                *
            return 0;
    }
    
```

| | | |
|--------------|---|----------|
| \mathbb{X} | \mathbb{V} | Ω |
| x | 42 | α |
| p | NULL | β |
| Φ | $(\alpha > 0) \wedge (\beta = \text{NULL})$ | |



$(\alpha > 0) \wedge (\beta \neq \text{NULL})$ is **SAT**
 when $(x, p) = (42, 0xA0BF : \boxed{0} \boxed{\text{NULL}})$

```

class Node {
    int data;
    Node* next;
};

void f(int x, Node *p) {
    *
    if (x > 0)

        if (p != NULL)

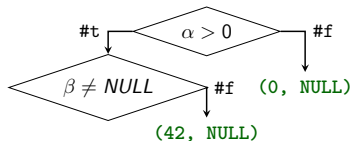
            if (x*2+1 == p->data)

                if (p->next == p)

                    ERROR;

    return 0;
}
    
```

| | | |
|--------------|-------------------|----------|
| \mathbb{X} | \mathbb{V} | Ω |
| x | 42 | α |
| p | 0xA0BF : 0 NULL | β |
| Φ | true | |




```

class Node {
    int data;
    Node* next;
};

void f(int x, Node *p) {

    if (x > 0)

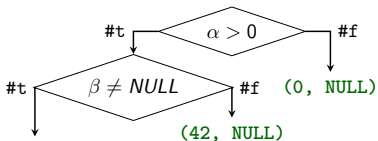
        if (p != NULL)
            *
            if (x*2+1 == p->data)

                if (p->next == p)

                    ERROR;

        return 0;
}
    
```

| | | | |
|--------------|--|----------|----------|
| \mathbb{X} | \mathbb{V} | | Ω |
| x | 42 | | α |
| p | 0xA0BF : | 0 NULL | β |
| Φ | $(\alpha > 0) \wedge (\beta \neq \text{NULL})$ | | |



```

class Node {
    int data;
    Node* next;
};

void f(int x, Node *p) {

    if (x > 0)

        if (p != NULL)

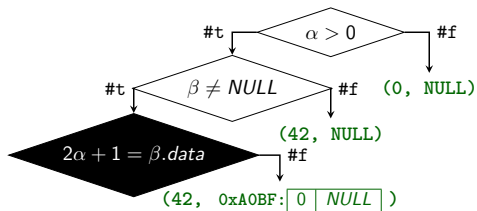
            if (x*2+1 == p->data)

                if (p->next == p)

                    ERROR;

    *
    return 0;
}
    
```

| | | |
|--------------|--|----------|
| \mathbb{X} | \mathbb{V} | Ω |
| x | 42 | α |
| p | 0xA0BF : 0 NULL | β |
| Φ | $(\alpha > 0) \wedge (\beta \neq \text{NULL}) \wedge (2\alpha + 1 \neq \beta.\text{data})$ | |



```

class Node {
    int data;
    Node* next;
};

void f(int x, Node *p) {

    if (x > 0)

        if (p != NULL)

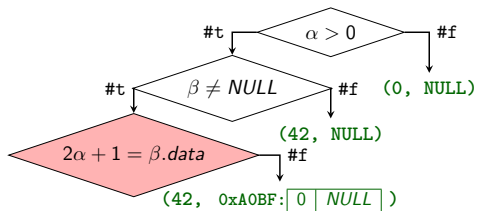
            if (x*2+1 == p->data)

                if (p->next == p)

                    ERROR;

                *
            return 0;
    }
    
```

| \mathbb{X} | \mathbb{V} | Ω |
|--------------|---|----------|
| x | 42 | α |
| p | 0xA0BF : 0 NULL | β |
| Φ | $(\alpha > 0) \wedge (\beta \neq \text{NULL}) \wedge$ $(2\alpha + 1 \neq \beta.\text{data})$ | |



$(\alpha > 0) \wedge (\beta \neq \text{NULL})$
 $\wedge (2\alpha + 1 = \beta.\text{data})$ is **SAT**

when $(x, p) = (1, 0xA0BF : \boxed{3} \mid \text{NULL})$

```

class Node {
    int data;
    Node* next;
};

void f(int x, Node *p) {
    *
    if (x > 0)

        if (p != NULL)

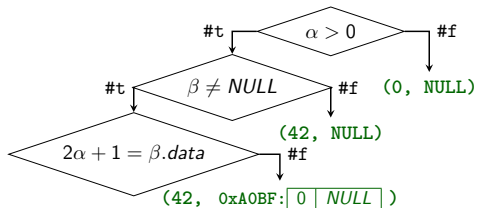
            if (x*2+1 == p->data)

                if (p->next == p)

                    ERROR;

    return 0;
}
    
```

| | | |
|--------------|-------------------|----------|
| \mathbb{X} | \mathbb{V} | Ω |
| x | 1 | α |
| p | 0xA0BF : 3 NULL | β |
| Φ | true | |



```

class Node {
    int data;
    Node* next;
};

void f(int x, Node *p) {

    if (x > 0)

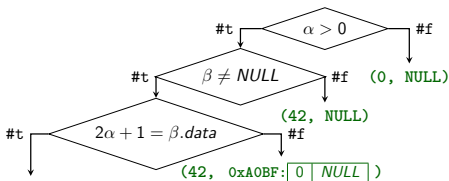
        if (p != NULL)

            if (x*2+1 == p->data)
                *
                if (p->next == p)

                    ERROR;

            return 0;
}
    
```

| | | | |
|--------------|--|----------|----------|
| \mathbb{X} | \mathbb{V} | | Ω |
| x | 1 | | α |
| p | 0xA0BF : | 3 NULL | β |
| Φ | $(\alpha > 0) \wedge (\beta \neq \text{NULL}) \wedge$ $(2\alpha + 1 = \beta.\text{data})$ | | |



```

class Node {
    int data;
    Node* next;
};

void f(int x, Node *p) {

    if (x > 0)

        if (p != NULL)

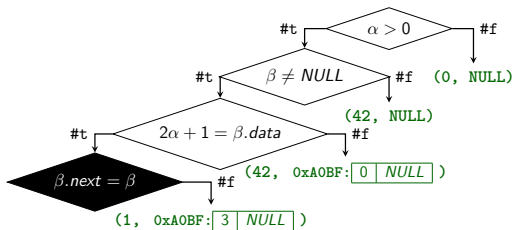
            if (x*2+1 == p->data)

                if (p->next == p)

                    ERROR;

    *
    return 0;
}
    
```

| | | | |
|--------------|--|----------|----------|
| \mathbb{X} | \mathbb{V} | | Ω |
| x | 1 | | α |
| p | 0xA0BF : | 3 NULL | β |
| Φ | $(\alpha > 0) \wedge (\beta \neq \text{NULL}) \wedge$ $(2\alpha + 1 = \beta.\text{data}) \wedge (\beta.\text{next} \neq \beta)$ | | |



```

class Node {
    int data;
    Node* next;
};

void f(int x, Node *p) {

    if (x > 0)

        if (p != NULL)

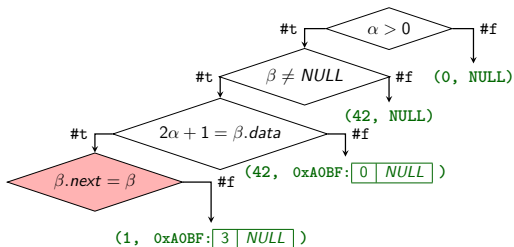
            if (x*2+1 == p->data)

                if (p->next == p)

                    ERROR;

    *
    return 0;
}
    
```

| | | | |
|--------------|--|----------|----------|
| \mathbb{X} | \mathbb{V} | | Ω |
| x | 1 | | α |
| p | 0xA0BF : | 3 NULL | β |
| Φ | $(\alpha > 0) \wedge (\beta \neq \text{NULL}) \wedge$ $(2\alpha + 1 = \beta.\text{data}) \wedge (\beta.\text{next} \neq \beta)$ | | |



$(\alpha > 0) \wedge (\beta \neq \text{NULL})$
 $\wedge (2\alpha + 1 = \beta.\text{data})$ is **SAT**

when $(x, p) = (1, 0xA0BF : \boxed{3} \mid \boxed{0xA0BF})$

```

class Node {
    int data;
    Node* next;
};

void f(int x, Node *p) {
    *
    if (x > 0)

        if (p != NULL)

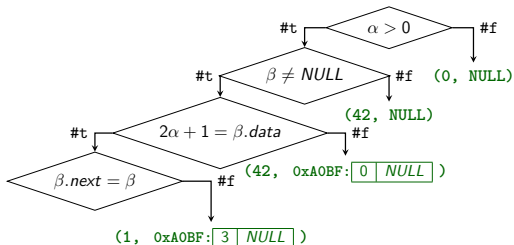
            if (x*2+1 == p->data)

                if (p->next == p)

                    ERROR;

    return 0;
}
    
```

| | | | |
|--------------|--------------|--------|----------|
| \mathbb{X} | \mathbb{V} | | Ω |
| x | 1 | | α |
| p | 0xA0BF : 3 | 0xA0BF | β |
| Φ | true | | |




```

class Node {
    int data;
    Node* next;
};

void f(int x, Node *p) {

    if (x > 0)

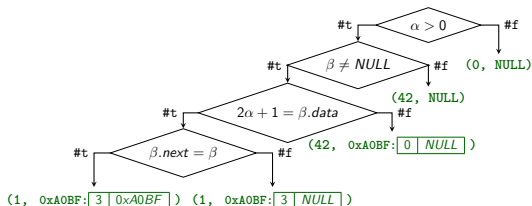
        if (p != NULL)

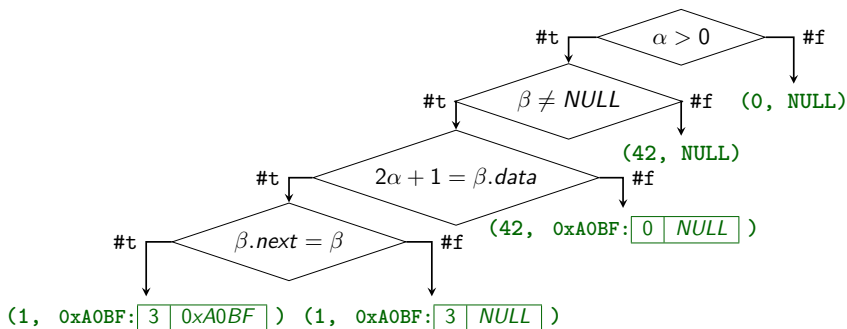
            if (x*2+1 == p->data)

                if (p->next == p)
                    *
                    ERROR;

    return 0;
}
    
```

| | | | |
|--------------|---|--------|----------|
| \mathbb{X} | \mathbb{V} | | Ω |
| x | 1 | | α |
| p | 0xA0BF : 3 | 0xA0BF | β |
| Φ | $(\alpha > 0) \wedge (\beta \neq \text{NULL}) \wedge$ $(2\alpha + 1 = \beta.\text{data}) \wedge (\beta.\text{next} = \beta)$ | | |



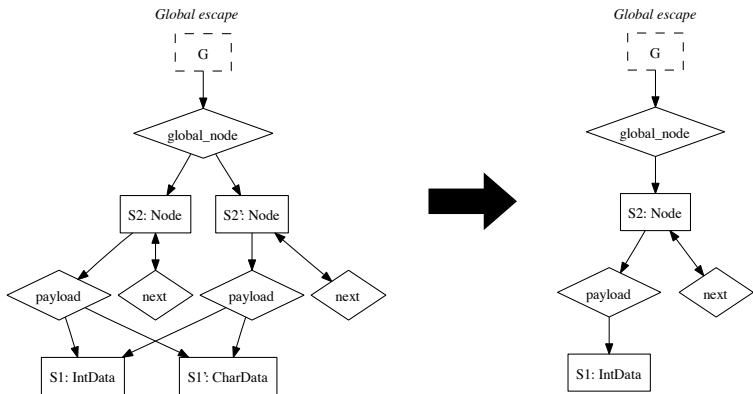


- [KLEE](#) – LLVM based DSE engine.
- [Jalangi2](#) – JavaScript dynamic analysis framework by Samsung.
- [S2E](#) – Platform for symbolic execution of binary code (x86, ARM).
- [CutEr](#) – Concolic unit testing tool for Erlang.

- Many efforts have combined dynamic and static analysis, yielding **blended** or **hybrid** analysis techniques.
- Some hybrid analyses use information recorded during dynamic executions of the program to improve the static-analysis precision. However, they are **generally unsound** because they rely on incomplete information.
- Some recent hybrid analyses have been proposed a **combined interpretation** to address the unsoundness issue, and it interchanges between **concrete** and **abstract interpretations** to improve the precision.

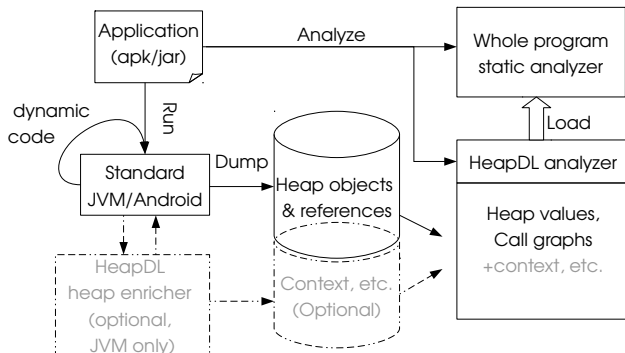
[ISSTA'07] Dufour et al., “Blended analysis for performance understanding of framework-based applications”

- **Escape Analysis** – A static analysis technique that determines whether an object can escape the scope of the method or a thread.



[OOPSLA'17] Grech et al., “Heaps Don’t Lie: Countering Unsoundness with Heap Snapshots”

- **Heap Snapshots** – A technique that records the whole-heap snapshots during program execution and then uses them to guide the static analysis. (especially for Android and JVM applications)

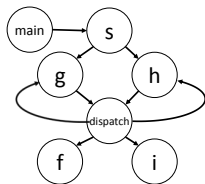


[PLDI'16] Briana M. Ren and Jeffrey S. Foster, "Just-in-time Static Type Checking for Dynamic Languages"

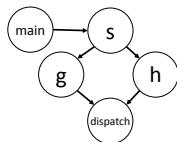
- **Just-in-time Static Type Checking** – A technique that type checks Ruby code even in the presence of meta-programming.
- It **dynamically gathers method type signatures** and utilizes them in **static type checking** of their body when they are invoked.

```
class Talk < ActiveRecord::Base
  belongs_to :owner, :class_name => "User"
  ....
  type :owner?, "(User) → %bool"
  def owner?(user)
    return owner == user
  end
end
```

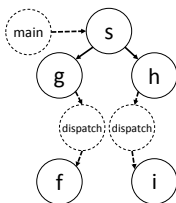
[POPL'18] J. Toman and D. Grossman, "Concerto: a framework for combined concrete and abstract interpretation"



(a) Sound but Imprecise Call-Graph



(b) Unsound Call-Graph

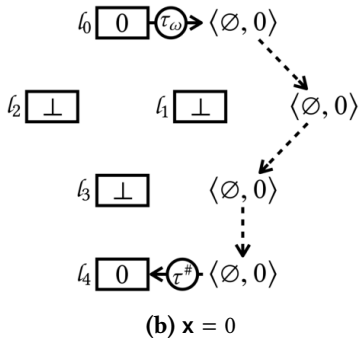


(c) Call-Graph with Concerto

[ESEC/FSE'21] J. Park et al., "Accelerating JavaScript Static Analysis via Dynamic Shortcuts"

- **Dynamic Shortcuts** – A technique to perform **sealed execution** (dynamic analysis) during the static analysis to accelerate and increase the precision of the analysis without losing soundness.

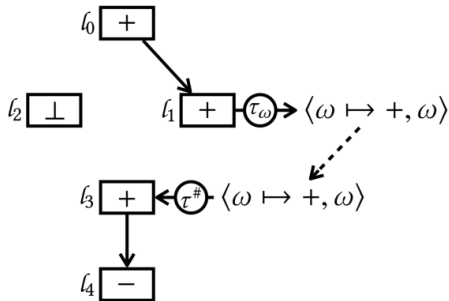
- ℓ_0 if ($x \geq 0$)
- ℓ_1 $x = x$;
- else
- ℓ_2 $x = -x$;
- ℓ_3 $x = -x$;
- ℓ_4



[ESEC/FSE'21] J. Park et al., "Accelerating JavaScript Static Analysis via Dynamic Shortcuts"

- **Dynamic Shortcuts** – A technique to perform **sealed execution** (dynamic analysis) during the static analysis to accelerate and increase the precision of the analysis without losing soundness.

- l_0 if ($x \geq 0$)
- l_1 $x = x$;
- else
- l_2 $x = -x$;
- l_3 $x = -x$;
- l_4

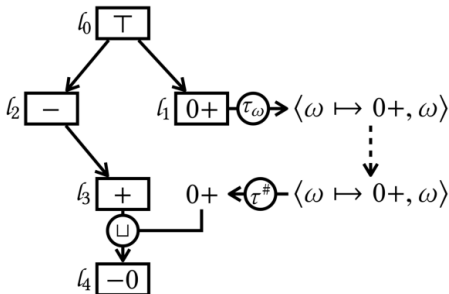


(c) $x > 0$

[ESEC/FSE'21] J. Park et al., "Accelerating JavaScript Static Analysis via Dynamic Shortcuts"

- **Dynamic Shortcuts** – A technique to perform **sealed execution** (dynamic analysis) during the static analysis to accelerate and increase the precision of the analysis without losing soundness.

- l_0 if ($x \geq 0$)
- l_1 $x = x$;
- else
- l_2 $x = -x$;
- l_3 $x = -x$;
- l_4



(d) $x \in \mathbb{N}$

1. Symbolic Execution

- Basic Idea

- Satisfiability Modulo Theories (SMT)

- Limitations of Symbolic Execution

2. Dynamic Symbolic Execution (DSE)

- Search Heuristics

- Example – Hash Function

- Example – Loops

- Example – Data Structures

- Realistic Implementation

- Other Hybrid Analysis Techniques

- Lecture for Dynamic Symbolic Execution by Prof. Mayur Naik at UPenn CIS.

<https://software-analysis-class.org/lectures/lecture14>

- [CSUR'18] Baldoni et al., “A Survey of Symbolic Execution Techniques”

<https://dl.acm.org/doi/abs/10.1145/3182657>

- Mutation Testing

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