Lecture 1 – Input Space Modeling

AAA705: Software Testing and Quality Assurance

Jihyeok Park

2026 Fall

Black-box Testing

- **Black-box testing** views the software as a **black box**: test data are derived from the **specification**, not from the code.

- We already know the input space is far too large to cover. So the real question is **which** inputs to pick – and before we can answer that, we need a **model** of the input space.

Contents

1. Modeling by Hand
    - Equivalence Partitioning (EP)
    - Boundary Value Analysis (BVA)
    - Category Partition Method (CPM)

2. Combinatorial Testing (CT)
    - Covering Arrays
    - Size of Covering Arrays
    - Construction and Coverage

3. Inferring the Input Model

Contents

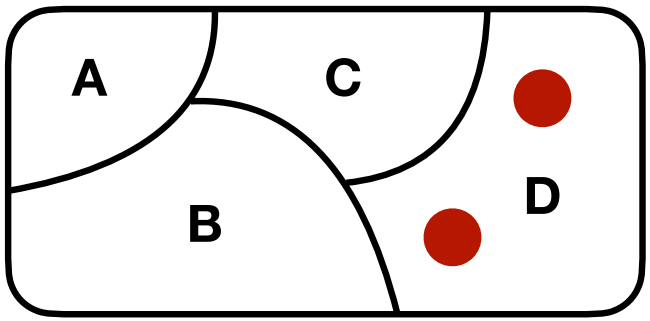
1. Modeling by Hand
   - Equivalence Partitioning (EP)
   - Boundary Value Analysis (BVA)
   - Category Partition Method (CPM)

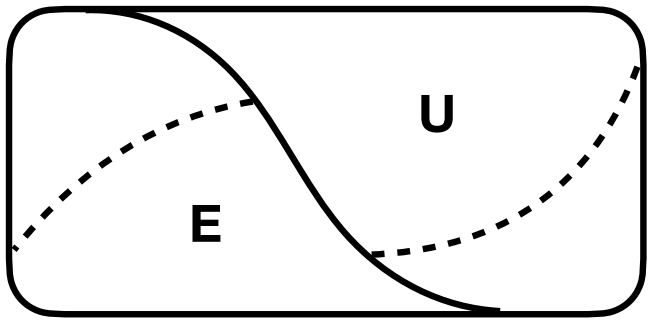
2. Combinatorial Testing (CT)
   - Covering Arrays
   - Size of Covering Arrays
   - Construction and Coverage

3. Inferring the Input Model

Equivalence Partitioning (EP)

Equivalence partitioning divides the input domain into **equivalence classes**, on the observation that the program should behave the **same way** for all members of a class.





- If one test case in an equivalence class reveals an error, the others in the **same class** likely reveal the **same error**. So we select **one test case** from each class.

- Then, how to define the classes? One way is to split the input domain into **expected (E)** (legal) and **unexpected (U)** (illegal) inputs, and then divide the expected inputs further.

Equivalence Partitioning (EP)

PLRG

Example

Consider a program that takes a **password** as input. The length of the password must be between 6 and 20 characters.

- We can divide the input domain into two equivalence classes:
  - $E = \{$ a password $p \mid 6 \leq |p| \leq 20\}$
  - $U = \{$ a password $p \mid |p| < 6 \vee |p| > 20\}$
- We can divide it more finely:
  - $E_1 = \{$ a password $p \mid 6 \leq |p| \leq 10\}$ for *weak passwords*
  - $E_2 = \{$ a password $p \mid 11 \leq |p| \leq 15\}$ for *medium-strength passwords*
  - $E_3 = \{$ a password $p \mid 16 \leq |p| \leq 20\}$ for *strong passwords*
  - $U_1 = \{$ a password $p \mid |p| < 6\}$ for *too short passwords*
  - $U_2 = \{$ a password $p \mid |p| > 20\}$ for *too long passwords*
- We can select one test case from each equivalence class.

$$I = \{p_1, p_2, p_3, p_4, p_5\}$$

such that $|p_1| = 7$, $|p_2| = 13$, $|p_3| = 18$, $|p_4| = 3$, $|p_5| = 40$

Equivalence Partitioning (EP)

PLRG

- There are **many ways** to partition the input domain.
- Even from the same equivalence classes, we can choose **different test cases**.
- **Effectiveness** may depend on the tester's **experience** and **intuition**.

  Partition testing can be better, worse, or the same as random testing, depending on how the partitioning is done.[1]

[1] **[TSE'91]** E. J. Weyuker, B. Jeng. "*Analyzing Partition Testing Strategies.*"

Off-by-one Errors – Looping Over Arrays

Logic errors often occur at the **boundaries** of the input domain, usually as **off-by-one** errors from misunderstanding the boundary conditions. Simple, but **very common**:

```
for (let i = 0; i < 10; i++) {
  /* body of the loop */
}
```

CORRECT

```
for (let i = 1; i < 10; i++) {
  /* body of the loop */
}
```

INCORRECT

```
for (let i = 0; i <= 10; i++) {
  /* body of the loop */
}
```

INCORRECT

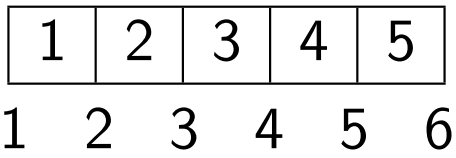
```
for (let i = 0; i < 9; i++) {
  /* body of the loop */
}
```

INCORRECT

Off-by-one Errors – Fencepost error

If you build a straight fence 15 meters long with posts spaced 3 meters apart, how many posts do you need?

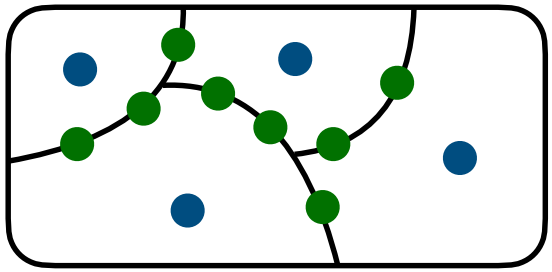
$15 \,/\, 3 = 5$ posts? No, you need 6 posts!



`linspace(a, b, n)` in MATLAB is a linear interpolation function that generates a row vector of n **points** instead of n **intervals** between a and b.

```
linspace(0, 10, 5) == [0, 2.5, 5, 7.5, 10]
                   != [0, 2, 4, 6, 8, 10]
```

Boundary Value Analysis (BVA)



- Equivalence Partitioning (EP)
- Boundary Value Analysis (BVA)

- **Boundary value analysis** selects test cases **at the boundaries** of the equivalence classes, because a program is more likely to fail there. It is used **together with** equivalence partitioning.
- For the password classes above (split at lengths $6, 10, 15, 20$), the boundaries give

$$|p_1| = 5 \quad |p_2| = 6 \quad |p_3| = 10 \quad |p_4| = 11$$
$$|p_5| = 15 \quad |p_6| = 16 \quad |p_7| = 20 \quad |p_8| = 21$$

  – **both sides** of every boundary, so a comparison written with $<$ instead of $\leq$ cannot survive.

Category Partition Method (CPM)

PLRG

- Most programs behave differently when they receive different **parameters** or are executed under different **environments**.

- **Category partition method (CPM)** is a black-box testing technique that systematically generates test cases by considering the combinations of the categories of the input domain.

  1. Analyze **specification**
  2. Identify **parameters** and **environments**
  3. Identify **categories** for each parameter and environment
  4. **Partition** categories into equivalence classes
  5. Identify **constraints**
  6. **Generate** test cases

Category Partition Method (CPM)

PLRG

Example

Unix command `grep` searches lines matching a pattern:

```
grep <pattern> <filename>
```

`grep park myfile` finds lines containing "`park`"; `grep "hello world" myfile` needs **quotes** for embedded spaces; `grep " said \"hello " myfile` needs a **backslash** for an embedded quotation mark.

① Analyze **specification** ② Identify **parameters** and **environments**

- **Parameters** – (1) `<pattern>`, (2) `<filename>`
- **Environments** – (3) file contents

Category Partition Method (CPM)

PLRG

③ Identify **categories** and ④ **Partition** them into equivalence classes

1. **Parameter** – `<pattern>`
   - **Size** – 0 / 1 / ≥ 2
   - **Quotation marks** – quoted (Q) / unquoted (U) / improper (I)
   - **Embedded spaces** – none (N) / single (S) / multiple (M)
   - **Embedded quotation marks** – none (N) / single (S) / multiple (M)
2. **Parameter** – `<filename>`
   - **Validity** – exists (E) / not exists (N) / omitted (O)
3. **Environment** – file contents
   - **Number of occurrences of the pattern** – 0 / 1 / ≥ 2
   - **Number of occurrences of the pattern in a line** – 0 / 1 / ≥ 2

Category Partition Method (CPM)

PLRG

⑤ Identify **constraints** – e.g. an **unquoted** pattern cannot have embedded spaces:

unquoted (U) $\not\Rightarrow\Leftarrow$ single (S) or multiple (M) **embedded spaces**

⑥ **Generate** test cases

- Pick one **test case** from each combination **satisfying** the constraints.

Size	Q	Space	Emb. Q	Valid	Occur	Occur
0	Q	N	N	E	0	0
1	U	S	S	N	1	1
≥ 2	I	M	M	O	≥ 2	≥ 2

```
# myfile has exactly one matching line
$ grep "hello world" myfile
she said hello world once
```

Contents

1. Modeling by Hand
   - Equivalence Partitioning (EP)
   - Boundary Value Analysis (BVA)
   - Category Partition Method (CPM)

2. Combinatorial Testing (CT)
   - Covering Arrays
   - Size of Covering Arrays
   - Construction and Coverage

3. Inferring the Input Model

Combinatorial Testing (CT)

- Testing all combinations is still **too expensive** because of the **combinatorial explosion**!
  - For example, we need 1,799,736,525 test cases required for the following airport system:

Airline	Destination	Departure Date	Return Date
79	171	365	365

- **Combinatorial testing (CT)** or **combinatorial interaction testing (CIT)** constructs test cases by considering the **interactions** between the parameters.

Covering Array (CA)

PLRG

Definition (Interaction)

For k parameters with v values each, a t-**way interaction** is a combination of values for t parameters.

Definition (Covering Array (CA))

A **covering array** $A = CA(N; t, k, v)$ is a $N \times k$ matrix such that every field is an element from the set $[0, v-1]$, and every t-way interaction is covered at least once by a row of A.

A	B	C
0	0	0
1	1	1

$\Rightarrow$

$CA(N = 4; t = 2, k = 3, v = 2)$

A	B	C
0	0	0
0	1	1
1	0	1
1	1	0

Mixed Covering Array (MCA)

Definition (Mixed Covering Array (MCA))

A **mixed covering array** $A = MCA(N; t, k, v = (v_1, v_2, \ldots, v_k))$ is a $N \times k$ matrix such that every field in the i-th column is an element from the set $[0, v_i - 1]$, and every t-way interaction is covered at least once by a row of A.

A	B	C
0	0	0
1	1	1
		2

$\Rightarrow$

$MCA(N = 6; t = 2, k = 3, v = (2, 2, 3))$

A	B	C
0	0	0
0	1	1
1	0	2
1	1	0
1	0	1
0	1	2

Constraint Mixed Covering Array (CMCA)

Definition (Constraint Mixed Covering Array (CMCA))

A **constraint mixed covering array** $A = CMCA(N; t, k, v = (v_1, v_2, \ldots, v_k), P)$ is a $N \times k$ matrix such that every field in the i-th column is an element from the set $[0, v_i - 1]$, and every **valid** t-way interaction is covered at least once by a row of A. We say that a t-way interaction is **valid** if it satisfies the predicate P.

A	B	C
0	0	0
1	1	1
		2

$\Rightarrow$

$CMCA(N = 5; t = 2, k = 3, v = (2, 2, 3), P)$

A	B	C
0	1	1
1	0	2
1	1	0
1	0	1
0	1	2

$P(x, y) = x + y > 0$

Combinatorial Testing (CT)

Definition (Combinatorial Testing (CT))

Combinatorial testing with strength t builds a test suite from a covering array $CA(N; t, k, v)$: each **row** becomes **one test case**. **Pairwise testing** is the special case $t = 2$.

Pairwise testing produces 5 test cases for the following system:

A	B	C
0	0	0
1	1	1
		2

$\Rightarrow$

$CMCA(N = 5; t = 2, k = 3, v = (2, 2, 3), P)$

A	B	C
0	1	1
1	0	2
1	1	0
1	0	1
0	1	2

$P(x, y) = x + y > 0$

Why Does Pairwise Testing Work?

A bug that sat inside `java.util.Arrays.binarySearch` for **nine years**.[2]

```
int mid = (low + high) / 2;   // int tops out at 2,147,483,647
```

	target in **front half**	target in **back half**
small array	ok	ok
huge array ($> 2^{30}$)	ok	**crash**

Only there do **both** grow large: `low` = 1,073,741,820, `high` = 2,147,483,639, so

$$\texttt{low + high} = 3{,}221{,}225{,}459 \longrightarrow -1{,}073{,}741{,}837 \longrightarrow \texttt{mid} < 0$$

Either one alone stays in range. It took **both** – which is why nobody hit it for nine years.

[2] **[Google'06]** J. Bloch. "*Nearly All Binary Searches and Mergesorts Are Broken.*" (also in *Programming Pearls*, 1986)

Failure-Triggering Fault Interaction (FTFI)

PLRG

Definition (FTFI Number)

The **FTFI number** of a fault is how many parameters must **all** be set to specific values to trigger the failure.

`binarySearch`'s fault: **FTFI = 2** $\Longrightarrow$ **pairwise finds it**

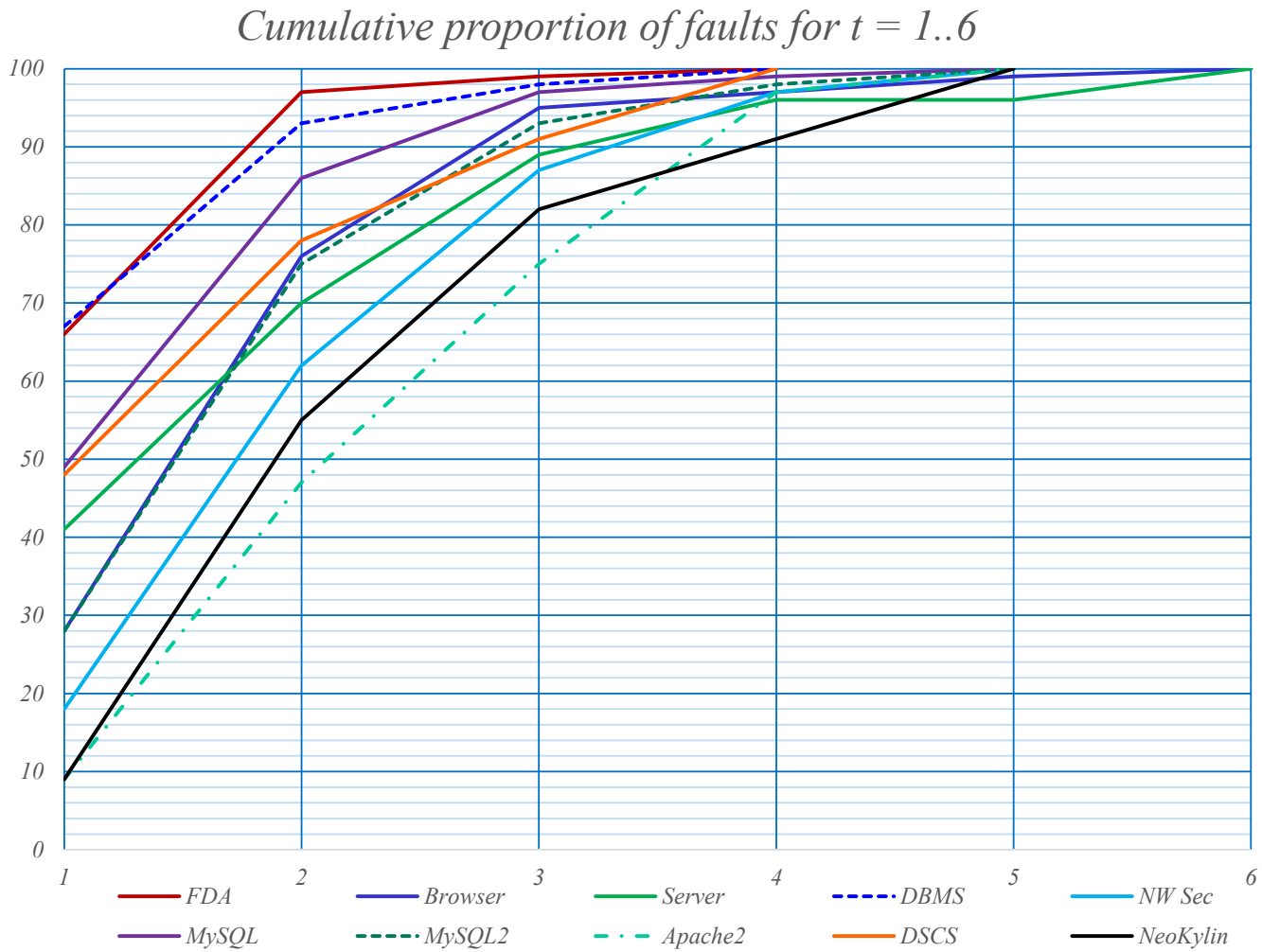
Kuhn et al. counted FTFI numbers in **real fault reports**, and the study has since been repeated across many systems.[3]

> **The Interaction Rule.** *Almost all faults have a* **small** *FTFI number, and in every system measured,* **none** *exceeded* $4 \sim 6$.

[3] **[TSE'04]** D. R. Kuhn, D. R. Wallace, A. M. Gallo. "*Software Fault Interactions and Implications for Software Testing.*"

Fault Detection Effectiveness

x-**axis**: strength t $(1 \rightarrow 6)$ y-**axis**: cumulative % of faults found



"Combinatorial Methods in Software Testing" by Rick Kuhn, NIST, 2019.

Lower Bound – $N \geq v^t$

The interaction rule says a **small** t is enough. But what does covering **all** t-way interactions **cost**? Bound N from below first.

Pick **any** t of the k columns. Together they have

$$\underbrace{v \times v \times \cdots \times v}_{t \text{ columns}} = v^t \quad \text{value combinations,}$$

and one **row** shows exactly **one** of them. All v^t must appear:

$$\boxed{N \geq v^t}$$

For example, in $CA(N{=}4;\ t{=}2,\ k{=}3,\ v{=}2)$, any two columns need all $2^2 = 4$ pairs, one row each – and $N = 4$ sits exactly on the floor:

A	B	C
0	0	0
0	1	1
1	0	1
1	1	0

$(A, B):\ \ (0,0)\ \ (0,1)\ \ (1,0)\ \ (1,1)$

The Strength t Is What Costs

The bound is v^t, so every $+1$ in strength multiplies the floor by v:

strength t	2	3	4	5	6
v^t for $v = 4$	16	64	256	1,024	4,096

- Recall the interaction rule: the **returns diminish** beyond $t = 2 \sim 3$, while this **cost multiplies**.
- That is why tools and standards settle at $t = 2$ or 3, and $t = 4 \sim 6$ is reserved for **safety-critical** systems where the last few percent matter.

But the Parameter Count k Barely Matters

Look again at the bound: $N \geq v^t$ has **no** k **in it**. Adding parameters never raises the floor.

Exact minimum pairwise test counts for **binary** parameters ($t = 2$, $v = 2$) bear that out:

parameters k	3	4	10	15
exhaustive 2^k	8	16	1,024	32,768
pairwise N	**4**	**5**	**6**	**7**

- k grows **five-fold**; N grows by **three**, while exhaustive testing grows by a factor of **4,096**.
- So it stays far below exhaustive – but **how far**? For that we need an **upper** bound.

Upper Bound – a Random Matrix Works

Fill an $N \times k$ matrix **at random**. Take $k = 10$ parameters with $v = 3$ values each, and $t = 2$.

Worry about **one** interaction – columns A, D must somewhere show $(0, 1)$:

$$\Pr[\text{a row shows it}] = \underbrace{\tfrac{1}{3} \times \tfrac{1}{3}}_{t \text{ columns}} = \tfrac{1}{9} = \frac{1}{v^t}$$

$$\Pr[\text{no row does}] = \left(\tfrac{8}{9}\right)^N = \left(1 - \frac{1}{v^t}\right)^N$$

There are $\binom{10}{2} \times 3^2 = 45 \times 9 = 405$ interactions $\left(= \binom{k}{t} v^t\right)$. Let X count how many end up **uncovered**; adding the 405 chances gives

$$\mathbb{E}[X] = 405 \times \left(\tfrac{8}{9}\right)^N \qquad \left(= \binom{k}{t} v^t \left(1 - \tfrac{1}{v^t}\right)^N\right)$$

Upper Bound – How Many Rows Suffice?

Raise N until the **average** drops below 1:

N	50	51	52	53
$\mathbb{E}[X]$	1.122	0.997	0.886	0.788
	$\geq \mathbf{1}$	< 1	< 1	< 1

The moment $\mathbb{E}[X] < 1$, **not every** matrix can have an uncovered interaction – so at least one has **zero**.

$$\Longrightarrow \quad N = \mathbf{51} \text{ already } \mathbf{guarantees} \text{ a covering array exists}$$

- At $N = 50$ the argument says **nothing** – not that none exists, only that it cannot decide.
- We proved existence by **averaging**, never by constructing. This is the **probabilistic method**.

Upper Bound – a Closed Form

$$\mathbb{E}[X] \;=\; \binom{k}{t} v^t \left(1 - \frac{1}{v^t}\right)^N \;\le\; k^t v^t \, e^{-N/v^t}$$

because $\binom{k}{t} \le k^t$ and $1 - x \le e^{-x}$ at $x = 1/v^t$. Both make it **larger**, so it still bounds.

$$k^t v^t \, e^{-N/v^t} \;<\; 1 \qquad \Longleftrightarrow \qquad \boxed{N \;>\; t\, v^t \ln(kv)}$$

$$\text{here} \quad t\, v^t \ln(kv) \;=\; 2 \cdot 9 \cdot \ln 30 \;\approx\; 62$$

$$\underbrace{16}_{\text{greedy finds}} \;<\; \underbrace{51}_{\mathbb{E}[X]<1} \;<\; \underbrace{62}_{\text{after both relaxations}}$$

The argument only proves 51 is **enough** – it never claimed to be tight. Each relaxation then gave away a little more.

What the Two Bounds Mean

$$\underbrace{v^t}_{\text{unavoidable}} \;\le\; N \;\le\; \underbrace{O\big(t\, v^t \log(kv)\big)}_{k \text{ enters only here}}$$

For $k = 20$ parameters with $v = 15$ values each:

$$\underbrace{225}_{v^t} \;\le\; \underbrace{441}_{\text{greedy finds}} \;\le\; \underbrace{2{,}567}_{t\, v^t \ln(kv)} \qquad \left(15^{20} \approx 3.3 \times 10^{23}\right)$$

- Add configuration options **almost for free**.
- Raise the strength **never** for free.

Finding a Minimum Covering Array

Existence is not construction. The bound says a small array is out there; it does not hand us one.

- Deciding whether a covering array of a given size exists is **NP-complete**, shown for the binary case by Seroussi and Bshouty.[4]
- So we give up on **optimal** and settle for two practical routes:
  - **Greedy** – fast, any parameters, not optimal. It is **set cover**: keep taking the test that covers the most still-uncovered interactions.
  - **Meta-heuristic** – smaller arrays, much slower.

[4] **[IEEE Trans. Inf. Theory'88]** G. Seroussi, N. H. Bshouty. "*Vector Sets for Exhaustive Testing of Logic Circuits.*"

Greedy Algorithm – IPOG Strategy

PLRG

- Since finding a **minimum** covering array is NP-complete, we settle for a **polynomial time** greedy algorithm.

- Let's learn the greedy algorithm called **IPOG (In-Parameter-Order-General)**[5] that generates a covering array with a strength t. It is not optimal but practical.

[5] **[ECBS'07]** Y. Lei, R. Kacker, D. R. Kuhn, V. Okun, J. Lawrence. "*IPOG: A General Strategy for T-Way Software Testing.*"

Greedy Algorithm – IPOG Strategy

PLRG

1. Initialize **test set** ts to be an empty set.
2. **Parameters** are P_1, P_2, ..., P_k.
3. Add a test into ts for **all interactions of the first t parameters**.
4. `for` $(i = t + 1;\ i \leq n;\ i{+}{+})$
   1. Let π be the **set of t-way interactions** involving **parameter** P_i and $t - 1$ parameters among the first $i - 1$ parameters.
   2. (**Horizontal Growth**) `for` (test $\gamma = (v_1, v_2, \cdots, v_{i-1}) \in ts$)
      1. **Choose** a value v_i of P_i
      2. **Replace** γ with $\gamma' = (v_1, v_2, \cdots, v_{i-1}, v_i)$ such that γ' covers the most number of interactions in π.
      3. **Remove** from π the interaction covered by γ'.
   3. (**Vertical Growth**) `for` (interaction $\alpha \in \pi$)
      1. `if` ($\exists$ a test covers α) **Remove** α from π.
      2. `else if` (possible) **Change** an existing test
      3. `else` **Add** a new test to cover α and **Remove** it from π.

Greedy Algorithm – IPOG Strategy

PLRG

1. Initialize **test set** *ts* to be an empty set.
2. **Parameters** are P_1, P_2, ..., P_k.
3. Add a test into *ts* for **all interactions of the first** t **parameters**.

Greedy Algorithm – IPOG Strategy

Example

We want to **pairwise testing** for the following system:

P_1	P_2	P_3	P_4
0	0	0	0
1	1	1	1
		2	2

Adding all combinations of values between the first 2 parameters:

$ts =$

P_1	P_2
0	0
0	1
1	0
1	1

Greedy Algorithm – IPOG Strategy

PLRG

1. Initialize **test set** *ts* to be an empty set.
2. **Parameters** are P_1, P_2, ..., P_k.
3. Add a test into *ts* for **all interactions of the first** t **parameters**.
4. `for` $(i = t + 1;\ i \leq n;\ i$++$)$ (**Horizontal Growth**)
   1. Let π be the **set of** t**-way interactions** involving **parameter** P_i and $t - 1$ parameters among the first $i - 1$ parameters.

Greedy Algorithm – IPOG Strategy

Set π as pairs to cover involving P_3:

$\pi =$

P_1	P_2	P_3
0		0
	0	0
0		1
	0	1
0		2
	0	2
1		0
	1	0
1		1
	1	1
1		2
	1	2

Greedy Algorithm – IPOG Strategy

PLRG

1. Initialize **test set** ts to be an empty set.
2. **Parameters** are P_1, P_2, ..., P_k.
3. Add a test into ts for **all interactions of the first t parameters**.
4. `for` $(i = t + 1;\ i \leq n;\ i$++$)$ (**Horizontal Growth**)
   1. Let π be the **set of t-way interactions** involving **parameter** P_i and $t - 1$ parameters among the first $i - 1$ parameters.
   2. `for` (test $\gamma = (v_1, v_2, \cdots, v_{i-1}) \in ts$)
      1. **Choose** a value v_i of P_i
      2. **Replace** γ with $\gamma' = (v_1, v_2, \cdots, v_{i-1}, v_i)$ such that γ' covers the most number of interactions in π.
      3. **Remove** from π the interaction covered by γ'.

Greedy Algorithm – IPOG Strategy

Adding values for P_3 in ts:

$ts =$

P_1	P_2	P_3
0	0	
0	1	
1	0	
1	1	

$\pi =$

P_1	P_2	P_3
0		0
	0	0
0		1
	0	1
0		2
	0	2
1		0
	1	0
1		1
	1	1
1		2
	1	2

Greedy Algorithm – IPOG Strategy

Adding values for P_3 in ts:

$ts =$

P_1	P_2	P_3
0	0	0
0	1	
1	0	
1	1	

$\pi =$

P_1	P_2	P_3
0		0
	0	0
0		1
	0	1
0		2
	0	2
1		0
	1	0
1		1
	1	1
1		2
	1	2

Greedy Algorithm – IPOG Strategy

Adding values for P_3 in ts:

$ts =$

P_1	P_2	P_3
0	0	0
0	1	1
1	0	
1	1	

$\pi =$

P_1	P_2	P_3
0		0
	0	0
0		1
	0	1
0		2
	0	2
1		0
	1	0
1		1
	1	1
1		2
	1	2

Greedy Algorithm – IPOG Strategy

Adding values for P_3 in ts:

$ts =$

P_1	P_2	P_3
0	0	0
0	1	1
1	0	1
1	1	

$\pi =$

P_1	P_2	P_3
0		0
	0	0
0		1
	0	1
0		2
	0	2
1		0
	1	0
1		1
	1	1
1		2
	1	2

Greedy Algorithm – IPOG Strategy

Adding values for P_3 in ts:

$ts =$

P_1	P_2	P_3
0	0	0
0	1	1
1	0	1
1	1	0

$\pi =$

P_1	P_2	P_3
0		0
	0	0
0		1
	0	1
0		2
	0	2
1		0
	1	0
1		1
	1	1
1		2
	1	2

Greedy Algorithm – IPOG Strategy

PLRG

1. Initialize **test set** ts to be an empty set.
2. **Parameters** are P_1, P_2, ..., P_k.
3. Add a test into ts for **all interactions of the first t parameters**.
4. `for` $(i = t + 1;\ i \leq n;\ i{+}{+})$
   1. Let π be the **set of t-way interactions** involving **parameter** P_i and $t - 1$ parameters among the first $i - 1$ parameters.
   2. (**Horizontal Growth**) `for` (test $\gamma = (v_1, v_2, \cdots, v_{i-1}) \in ts$)
      1. **Choose** a value v_i of P_i
      2. **Replace** γ with $\gamma' = (v_1, v_2, \cdots, v_{i-1}, v_i)$ such that γ' covers the most number of interactions in π.
      3. **Remove** from π the interaction covered by γ'.
   3. (**Vertical Growth**) `for` (interaction $\alpha \in \pi$)
      1. `if` ($\exists$ a test covers α) **Remove** α from π.
      2. `else if` (possible) **Change** an existing test
      3. `else` **Add** a new test to cover α and **Remove** it from π.

Greedy Algorithm – IPOG Strategy

Extending ts:

$ts =$

P_1	P_2	P_3
0	0	0
0	1	1
1	0	1
1	1	0

$\pi =$

P_1	P_2	P_3
0		0
	0	0
0		1
	0	1
0		2
	0	2
1		0
	1	0
1		1
	1	1
1		2
	1	2

Greedy Algorithm – IPOG Strategy

Extending ts:

$ts =$

P_1	P_2	P_3
0	0	0
0	1	1
1	0	1
1	1	0
0	0	2

$\pi =$

P_1	P_2	P_3
0		0
	0	0
0		1
	0	1
0		2
	0	2
1		0
	1	0
1		1
	1	1
1		2
	1	2

Greedy Algorithm – IPOG Strategy

Extending ts:

$ts =$

P_1	P_2	P_3
0	0	0
0	1	1
1	0	1
1	1	0
0	0	2
1	1	2

$\pi =$

P_1	P_2	P_3
0		0
	0	0
0		1
	0	1
0		2
	0	2
1		0
	1	0
1		1
	1	1
1		2
	1	2

Greedy Algorithm – IPOG Strategy

The loop **continues** with P_4. **Horizontal growth** extends all six rows; of the $6 + 6 + 9 = 21$ new interactions, 6 are left over.

$ts =$

P_1	P_2	P_3	P_4
0	0	0	0
0	1	1	1
1	0	1	2
1	1	0	0
0	0	2	1
1	1	2	2

$\pi =$

P_1	P_2	P_3	P_4
0			2
1			1
		0	1
		0	2
		1	0
		2	0

Greedy Algorithm – IPOG Strategy

Vertical growth. A new row fixes only the two positions its interaction names; the rest are **don't cares**.

P_1	P_2	P_3	P_4
0	–	–	2
1	–	–	1
–	–	1	0
–	–	2	0

no new row for two

		0	2
		0	1

fill the don't cares →

P_1	P_2	P_3	P_4
0	0	0	2
1	0	0	1
0	0	1	0
0	0	2	0

Six interactions, **four** rows. Ten in total, against a lower bound of $3 \times 3 = 9$.

Greedy vs. Meta-heuristic

IPOG is an algorithm *for covering arrays*. A **meta-heuristic** is tied to no problem at all: represent the candidates, score them, and it searches. Hence *meta*.

Here that is **simulated annealing**, one of the search methods used in **Search-Based Software Testing (SBST)**.[6]

Size comparison

Subject	Greedy	Meta-heuristic
SPIN-S	27	19
SPIN-V	42	36
GCC	24	21
Apache	42	32
Bugzilla	21	16

Time (sec.) comparison

Subject	Greedy	Meta-heuristic
SPIN-S	0.2	8.6
SPIN-V	11.3	102.1
GCC	204	1902.0
Apache	76.4	109.1
Bugzilla	1.9	9.1

[6] **[SSBSE'09]** B. J. Garvin, M. B. Cohen, M. B. Dwyer. "*An Improved Meta-heuristic Search for Constrained Interaction Testing.*"

Measuring an Existing Suite

The **reverse** question: for a suite **not** built by combinatorial testing – inherited tests, fuzzer output, a conformance suite – how much t-way coverage does it **already** have? NIST's **CCM** tool measures exactly this.[7]

total – fraction of **all** t-way interactions covered

simple – fraction of parameter t-subsets covered **completely**

Take our $CA(N{=}4;\ t{=}2,\ k{=}3,\ v{=}2)$ and **drop one row**:

A	B	C
0	0	0
0	1	1
1	0	1

(A, B) : missing $(1, 1)$
(A, C) : missing $(1, 0)$
(B, C) : missing $(1, 0)$

$\text{total} = 9/12 = \mathbf{75\%}$
$\text{simple} = 0/3 = \mathbf{0\%}$

75% sounds healthy, yet **not one** parameter pair is fully covered – report only the first number and the suite looks far better than it is.

[7] **[ICSTW'13]** D. R. Kuhn, R. N. Kacker, Y. Lei. "*Combinatorial Coverage Measurement Concepts and Applications.*"

Contents

1. Modeling by Hand
    - Equivalence Partitioning (EP)
    - Boundary Value Analysis (BVA)
    - Category Partition Method (CPM)

2. Combinatorial Testing (CT)
    - Covering Arrays
    - Size of Covering Arrays
    - Construction and Coverage

3. Inferring the Input Model

Everything So Far Assumed a Model

parameters and values $\longrightarrow$ covering array $\longrightarrow$ tests

- But **who wrote** the parameters and values? So far: a **human**, using *EP*, *BVA*, and *CPM*.
- Can we **infer** the model instead?

inferred from	**technique (paper)**	**what you get (model)**
– (written by hand)	*EP* / *BVA* / *CPM*	parameters, values
source code (`#ifdef`)	Nadi et al. ICSE'14	configuration constraints
documentation	DocTer ISSTA'22	input constraints
examples + an oracle	GLADE PLDI'17 / Arvada ASE'21	a grammar
a seed corpus	Learn&Fuzz ASE'17	a statistical model

From Source Code – Configuration Constraints

`#ifdef X` asks **"was option X selected?"** So the configuration options **are** our parameters, and `on`/`off` their values.

```
#ifndef Y
void foo() { ... }
#endif
#ifdef X
void bar() { foo(); }
#endif
```

- Turn on both and `foo` is **called but never declared** – it does not compile. So every valid configuration satisfies

$$X \rightarrow \neg Y$$

- **Nadi et al.**[8] reads them out of the code – the **constraints** of a CMCA, written by nobody.

[8] **[ICSE'14]** S. Nadi, T. Berger, C. Kästner, K. Czarnecki. "*Mining Configuration Constraints: Static Analyses and Empirical Results.*"

From Documentation – Input Constraints

PLRG

An API's documentation already **is** an input model written in English.

> `tf.nn.max_pool3d(input, ksize, strides, padding, ...)`
>
> `input`: A **5-D Tensor** of the format specified by `data_format`.
>
> `padding`: A **string**, either **'VALID'** or **'SAME'**.

parameter	**extracted constraint**
`input`	structure $=$ tensor, ndim $= 5$
`padding`	dtype $=$ string, values $= \{$`'VALID'`, `'SAME'`$\}$

The second row is a **parameter with two values** – exactly what *CPM* gave us by hand. **DocTer**[9] reads it off the prose instead.

[9] **[ISSTA'22]** D. Xie, Y. Li, M. Kim, H. V. Pham, L. Tan, X. Zhang, M. W. Godfrey. "*DocTer: Documentation-Guided Fuzzing for Testing Deep Learning API Functions.*"

From Examples – Guess, Then Ask the Oracle

Fully **black-box**. All we have is **one** valid input and an oracle answering "**is this valid?**":

$$\texttt{seed} \;=\; \texttt{<a>hi} \qquad \underbrace{A \to (\texttt{a} + \cdots + \texttt{z} + \texttt{<a>}A\texttt{})^*}_{\text{what the oracle accepts – } \textbf{hidden from us}}$$

Guess a generalization, then **ask the oracle** about the strings it newly allows. Keep it only if they all pass.

generalize	**candidate**	**ask the oracle**
repeat `hi`	$(\texttt{<a>}(\texttt{hi})^*\texttt{})^*$	`<a></a>`, `<a>hihi</a>` ✓
split `hi`	$(\texttt{<a>}(\texttt{h}+\texttt{i})^*\texttt{})^*$	`<a>h</a>`, `<a>i</a>` ✓

Repeating this until nothing more survives gives a **regular expression**.

From Examples – From a Regex to a Grammar

That regex puts <a>... **next to** each other, but never **inside** one another. Name each $(\ )^*$:

$$A_1 \to (\texttt{<a>}\, A_2 \,\texttt{})^* \qquad\qquad A_2 \to (\texttt{h} + \texttt{i})^*$$

Is A_2 the same as A_1? ask <a><a>hi<a>hi ✓

Yes – the text slot can hold a whole <a>.... One name, one **grammar**:

$$A \to (\texttt{<a>}\, A \,\texttt{})^* \;\mid\; (\texttt{h} + \texttt{i})^*$$

Unbounded nesting, out of one nine-character example. A last pass widens h+i to a+$\cdots$+z – and that is exactly the hidden grammar.

GLADE[10] is the above; **Arvada**[11] pushes it to deeply recursive grammars.

[10] **[PLDI'17]** O. Bastani, R. Sharma, A. Aiken, P. Liang. "*Synthesizing Program Input Grammars.*"

[11] **[ASE'21]** N. Kulkarni, C. Lemieux, K. Sen. "*Learning Highly Recursive Input Grammars.*"

From a Corpus – Learning a Statistical Model

Learn&Fuzz[12]: no oracle, no grammar – just **examples**. 63,000 PDF objects, from a format specified in **1,300 pages**.

```
2 0 obj
<< /Type /Pages  /Kids [ 3 0 R ]  /Count 1 >>
endobj
```

Train a network to predict the **next character**, then sample from `obj` until `endobj`:

$$\texttt{2 0 obj << /Ty} \longrightarrow \Pr[\text{next character}] \longrightarrow \texttt{p}$$

That is the whole model: **training an RNN on a corpus already *is* input space modeling** – no rule is ever written down.

A perfect model makes a poor fuzzer, so **SampleFuzz** emits the **least** likely character exactly where the model is **most confident**.

[12] **[ASE'17]** P. Godefroid, H. Peleg, R. Singh. "*Learn&Fuzz: Machine Learning for Input Fuzzing.*"

Summary

1. Modeling by Hand
   - Equivalence Partitioning (EP)
   - Boundary Value Analysis (BVA)
   - Category Partition Method (CPM)

2. Combinatorial Testing (CT)
   - Covering Arrays
   - Size of Covering Arrays
   - Construction and Coverage

3. Inferring the Input Model

Next Lecture

- Random Testing and Fuzzing

Jihyeok Park
jihyeok_park@korea.ac.kr
https://plrg.korea.ac.kr