Lecture 11 – Mutable Variables

COSE212: Programming Languages

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PLRG

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Recall

- Mutation makes it possible to **change the state** of a program by **updating the contents** of a data structure or a variable.
  - **Mutable data structures**
  - **Mutable variables**

- **Mutable Data Structures** – Mutable Boxes

- BFAE – FAE with Mutable Boxes
  - Evaluation with Memories

- In this lecture, we will learn **Mutable Variables**

- **MFAE – FAE with Mutable Variables**
  - Concrete and Abstract Syntax
  - Interpreter and Natural Semantics

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Mutable Variables

A **mutable variable** is a variable whose value can be changed after its initialization.

Let's define mutable variables in Scala:

```
// A mutable variable `x` of type `Int` with 1
var x: Int = 1
x + 2                // 1 + 2 == 3 : Int

// We can reassign a mutable variable `x`
x = 2                // x == 2
x + 2                // 2 + 2 == 4 : Int

// The function `f` is impure because it uses a mutable variable `y`
var y: Int = 1
def f(x: Int): Int = x + y
f(5)                 // 5 + 1 == 6 : Int
y = 3
f(5)                 // 5 + 3 == 8 : Int
```

Contents

MFAE – FAE with Mutable Variables

Now, let's extend FAE into MFAE to support **mutable variables**.

```
/* MFAE */
var x = 5;
x;          // 5
x = 8;
x           // 8
```

```
/* MFAE */
var y = 1;
var f = x => { x = x + y; x * x };
f(5);     // (5 + 1) * (5 + 1) = 36
y = 3;
f(5);     // (5 + 3) * (5 + 3) = 64
```

For MFAE, we need to extend **expressions** of FAE with

1. **mutable variables** (`var`) rather than immutable variables (`val`) (all variables, including parameters, are mutable in MFAE)

2. **assignment** (=) (right-associative: e.g., $x = y = e$ is equivalent to $x = (y = e)$)

3. **sequence** of expressions

Concrete Syntax

```
// expressions
<expr> ::= ...
         | "var" <id> "=" <expr> ";" <expr>
         | <id> "=" <expr>
         | <expr> ";" <expr>
```

For MFAE, we need to extend **expressions** of FAE with

1. **mutable variables** (`var`) rather than immutable variables (`val`) (all variables, including parameters, are mutable in MFAE)

2. **assignment** (=) (right-associative: e.g., $x = y = e$ is equivalent to $x = (y = e)$)

3. **sequence** of expressions

Abstract Syntax

Let's define the **abstract syntax** of MFAE in BNF:

$$
\begin{array}{llll}
\text{Expressions} & \mathbb{E} \ni e ::= \ldots & & \\
 & \mid \texttt{var}\ x\ \texttt{=}\ e;\ e & & (\texttt{Var}) \\
 & \mid x\ \texttt{=}\ e & & (\texttt{Assign}) \\
 & \mid e;\ e & & (\texttt{Seq})
\end{array}
$$

```
enum Expr:
  ...
  // mutable variable definition
  case Var(name: String, init: Expr, body: Expr)
  // variable assignment
  case Assign(name: String, expr: Expr)
  // sequence
  case Seq(left: Expr, right: Expr)
```

Contents

Evaluation with Memories

We can represent mutable variables by assigning different **addresses** to each variable in the environment and storing their values in the **memory**.

Let's see how to evaluate the following MFAE expression:

```
/* MFAE */                         *
var x = 5;
x;
x = 8;
x
```

$$\sigma = [\quad]$$

$\mathbb{A}$	:	a_0	a_1	a_2	a_3	...
M	=					...

Evaluation with Memories

We can represent mutable variables by assigning different **addresses** to each variable in the environment and storing their values in the **memory**.

Let's see how to evaluate the following MFAE expression:

```
/* MFAE */
var x = 5;                  *
x;
x = 8;
x
```

$$\sigma = [\; \texttt{x} \mapsto a_0 \;]$$

$\mathbb{A}$	:	a_0	a_1	a_2	a_3	$\ldots$
M	=	5				$\ldots$

Evaluation with Memories

We can represent mutable variables by assigning different **addresses** to each variable in the environment and storing their values in the **memory**.

Let's see how to evaluate the following MFAE expression:

```
/* MFAE */
var x = 5;
x;            /* 5 */      *
x = 8;
x
```

$$\sigma = [\; \texttt{x} \mapsto a_0 \;]$$

$\mathbb{A}$:	a_0	a_1	a_2	a_3	...
M =	5				...

Evaluation with Memories

We can represent mutable variables by assigning different **addresses** to each variable in the environment and storing their values in the **memory**.

Let's see how to evaluate the following MFAE expression:

```
/* MFAE */
var x = 5;
x;              /* 5 */
x = 8;                    *
x
```

$$\sigma = [\; \mathtt{x} \mapsto a_0 \;]$$

$\mathbb{A}$	:	a_0	a_1	a_2	a_3	$\ldots$
M	$=$	8				$\ldots$

We can represent mutable variables by assigning different **addresses** to each variable in the environment and storing their values in the **memory**.

Let's see how to evaluate the following MFAE expression:

```
/* MFAE */
var x = 5;
x;            /* 5 */
x = 8;
x             /* 8 */     *
```

$$\sigma = [\; \mathtt{x} \mapsto a_0 \;]$$

$\mathbb{A}$	:	a_0	a_1	a_2	a_3	$\ldots$
M	=	8				$\ldots$

Evaluation with Memories

Here is another MFAE expression:

```
/* MFAE */                *
var y = 1;
var f = x => {
  x = x + y;
  x * x
};
f(5);
y = 3;
f(5);
```

$$\sigma = [\quad]$$

$$\begin{array}{ccccccc} \mathbb{A} & : & a_0 & a_1 & a_2 & a_3 & \ldots \\ M & = & \square & \square & \square & \square & \ldots \end{array}$$

Example

Here is another MFAE expression:

```
/* MFAE */
var y = 1;                 *
var f = x => {
  x = x + y;
  x * x
};
f(5);
y = 3;
f(5);
```

$$\sigma = [\ y \mapsto a_0\ ]$$

$\mathbb{A}$	:	a_0	a_1	a_2	a_3	$\ldots$
M	$=$	1				$\ldots$

Example

Here is another MFAE expression:

```
/* MFAE */
var y = 1;
var f = x => {
  x = x + y;
  x * x
};                        *
f(5);
y = 3;
f(5);
```

$$\sigma = [\ \mathtt{y} \mapsto a_0,\ \mathtt{f} \mapsto a_1\ ]$$

$$\begin{array}{ccccccc} \mathbb{A} & : & a_0 & a_1 & a_2 & a_3 & \ldots \\ M & = & 1 & v & & & \ldots \end{array}$$

where $v = \langle \lambda \mathtt{x}.(\mathtt{x = x + y;\ x * x}), [\mathtt{y} \mapsto a_0] \rangle$

Example

Here is another MFAE expression:

```
/* MFAE */
var y = 1;
var f = x => {          *
  x = x + y;
  x * x
};
f(5);
y = 3;
f(5);
```

$$\sigma = [\ \mathtt{y} \mapsto a_0,\ \mathtt{x} \mapsto a_2\ ]$$

$\mathbb{A}$	:	a_0	a_1	a_2	a_3	$\ldots$
M	=	1	v	5		$\ldots$

where $v = \langle \lambda \mathtt{x}.(\mathtt{x = x + y;\ x * x}), [\mathtt{y} \mapsto a_0] \rangle$

Example

Here is another MFAE expression:

```
/* MFAE */
var y = 1;
var f = x => {
  x = x + y; /* 5 + 1 */     *
  x * x
};
f(5);
y = 3;
f(5);
```

$$\sigma = [\ y \mapsto a_0,\ x \mapsto a_2\ ]$$

$\mathbb{A}$	:	a_0	a_1	a_2	a_3	...
M	=	1	v	6		...

where $v = \langle \lambda x.(x = x + y;\ x * x), [y \mapsto a_0] \rangle$

Example

Here is another MFAE expression:

```
/* MFAE */
var y = 1;
var f = x => {
  x = x + y; /* 5 + 1 */
  x * x      /* 6 * 6 */     *
};
f(5);
y = 3;
f(5);
```

$$\sigma = [\; \texttt{y} \mapsto a_0,\; \texttt{x} \mapsto a_2 \;]$$

$\mathbb{A}$	:	a_0	a_1	a_2	a_3	...
M	=	1	v	6		...

where $v = \langle \lambda \texttt{x}.(\texttt{x = x + y; x * x}), [\texttt{y} \mapsto a_0] \rangle$

Example

Here is another MFAE expression:

```
/* MFAE */
var y = 1;
var f = x => {
  x = x + y;
  x * x
};
f(5);          /* 36 */      *
y = 3;
f(5);
```

$\sigma = [$

$\quad \mathtt{y} \mapsto a_0$

$\quad \mathtt{f} \mapsto a_1$

$]$

$\mathbb{A}$	:	a_0	a_1	a_2	a_3	$\ldots$
M	=	1	v	6		$\ldots$

where $v = \langle \lambda \mathtt{x}.(\mathtt{x = x + y;\ x * x}), [\mathtt{y} \mapsto a_0] \rangle$

Example

Here is another MFAE expression:

```
/* MFAE */
var y = 1;
var f = x => {
  x = x + y;
  x * x
};
f(5);          /* 36 */
y = 3;                  *
f(5);
```

$$\sigma = [\; \mathtt{y} \mapsto a_0 \;\; \mathtt{f} \mapsto a_1 \;]$$

$\mathbb{A}$	:	a_0	a_1	a_2	a_3	$\ldots$
M	=	3	v	6		$\ldots$

where $v = \langle \lambda \mathtt{x}.(\mathtt{x = x + y;\ x * x}), [\mathtt{y} \mapsto a_0] \rangle$

Example

Here is another MFAE expression:

```
/* MFAE */
var y = 1;
var f = x => {                    *
  x = x + y;
  x * x
};
f(5);          /* 36 */
y = 3;
f(5);
```

$$\sigma = [\ \mathtt{y} \mapsto a_0,\ \mathtt{x} \mapsto a_3\ ]$$

$\mathbb{A}$	:	a_0	a_1	a_2	a_3	...
M	=	3	v	6	5	...

where $v = \langle \lambda \mathtt{x}.(\mathtt{x = x + y;\ x * x}), [\mathtt{y} \mapsto a_0] \rangle$

Example

Here is another MFAE expression:

```
/* MFAE */
var y = 1;
var f = x => {
  x = x + y; /* 5 + 3 */     *
  x * x
};
f(5);        /* 36 */
y = 3;
f(5);
```

$$\sigma = [\; \mathtt{y} \mapsto a_0 \;\; \mathtt{x} \mapsto a_3 \;]$$

$\mathbb{A}$	:	a_0	a_1	a_2	a_3	$\ldots$
M	$=$	3	v	6	8	$\ldots$

where $v = \langle \lambda \mathtt{x}.(\mathtt{x = x + y;\; x * x}), [\mathtt{y} \mapsto a_0] \rangle$

Example

Here is another MFAE expression:

```
/* MFAE */
var y = 1;
var f = x => {
  x = x + y; /* 5 + 3 */
  x * x      /* 8 * 8 */      *
};
f(5);        /* 36 */
y = 3;
f(5);
```

$$\sigma = [\ \mathtt{y} \mapsto a_0,\ \mathtt{x} \mapsto a_3\ ]$$

$\mathbb{A}$	:	a_0	a_1	a_2	a_3	$\dots$
M	=	3	v	6	8	$\dots$

where $v = \langle \lambda \mathtt{x}.(\mathtt{x = x + y;\ x * x}), [\mathtt{y} \mapsto a_0] \rangle$

Example

Here is another MFAE expression:

```
/* MFAE */
var y = 1;
var f = x => {
  x = x + y;
  x * x
};
f(5);          /* 36 */
y = 3;
f(5);          /* 64 */        *
```

$$\sigma = [\ \mathtt{y} \mapsto a_0,\ \mathtt{f} \mapsto a_1\ ]$$

$\mathbb{A}$	:	a_0	a_1	a_2	a_3	$\ldots$
M	=	3	v	6	8	$\ldots$

where $v = \langle \lambda \mathtt{x}.(\mathtt{x = x + y;\ x * x}), [\mathtt{y} \mapsto a_0] \rangle$

Interpreter and Natural Semantics

PLRG

For MFAE, we need to 1) implement the **interpreter** with environments and **memories** by passing the updated memory in the result:

```
def interp(expr: Expr, env: Env, mem: Mem): (Value, Mem) = ???
```

```
type Env  = Map[String, Addr]
type Addr = Int
type Mem  = Map[Addr, Value]
```

```
enum Value:
  case NumV(n: BigInt)
  case CloV(p: String, b: Expr, e: Env)
```

and 2) define the **natural semantics** with environments and **memories** by passing the updated memory in the result:

$$\sigma, M \vdash e \Rightarrow v, M$$

Environments	σ	$\in \mathbb{X} \xrightarrow{\text{fin}} \mathbb{A}$	$(\texttt{Env})$
Addresses	a	$\in \mathbb{A}$	$(\texttt{Addr})$
Memories	M	$\in \mathbb{A} \xrightarrow{\text{fin}} \mathbb{V}$	$(\texttt{Mem})$

Mutable Variable

```
def interp(expr: Expr, env: Env, mem: Mem): (Value, Mem) = expr match
  ...
  case Var(name, init, body) =>
    val (iv, imem) = interp(init, env, mem)
    val addr = malloc(imem)
    interp(body, env + (name -> addr), imem + (addr -> iv))
```

$$\boxed{\sigma, M \vdash e \Rightarrow v, M}$$

$$\texttt{Var}\ \frac{a \notin \mathsf{Domain}(M_1) \qquad \begin{array}{c}\sigma, M \vdash e_1 \Rightarrow v_1, M_1 \\ \sigma[x \mapsto a], M_1[a \mapsto v_1] \vdash e_2 \Rightarrow v_2, M_2\end{array}}{\sigma, M \vdash \texttt{var}\ x\ \texttt{=}\ e_1\texttt{;}\ e_2 \Rightarrow v_2, M_2}$$

We learned one way to implement `malloc` in the previous lecture:

```
def malloc(mem: Mem): Addr = mem.keySet.maxOption.fold(0)(_ + 1)
```

Identifier Lookup

```
def interp(expr: Expr, env: Env, mem: Mem): (Value, Mem) = expr match
  ...
  case Id(name) => (mem(lookupId(env, name)), mem)

def lookupId(env: Env, name: String): Addr =
  env.getOrElse(name, error(s"free identifier: $name"))
```

$$\boxed{\sigma, M \vdash e \Rightarrow v, M}$$

$$\texttt{Id}\ \frac{x \in \mathsf{Domain}(\sigma)}{\sigma, M \vdash x \Rightarrow M(\sigma(x)), M}$$

Function Application

```
def interp(expr: Expr, env: Env, mem: Mem): (Value, Mem) = expr match
  ...
  case App(fun, arg) =>
    val (fv, fmem) = interp(fun, env, mem)
    fv match
      case CloV(param, body, fenv) =>
        val (av, amem) = interp(arg, env, fmem)
        val addr = malloc(amem)
        interp(body, fenv + (param -> addr), amem + (addr -> av))
      case _ =>
        error(s"not a function: ${fv.str}")
```

$$\boxed{\sigma, M \vdash e \Rightarrow v, M}$$

$$\texttt{App}\ \frac{\begin{array}{ll}\sigma, M \vdash e_1 \Rightarrow \langle \lambda x.e_3, \sigma' \rangle, M_1 & \sigma, M_1 \vdash e_2 \Rightarrow v_2, M_2 \\ a \notin \mathsf{Domain}(M_2) & \sigma'[x \mapsto a], M_2[a \mapsto v_2] \vdash e_3 \Rightarrow v_3, M_3\end{array}}{\sigma, M \vdash e_1(e_2) \Rightarrow v_3, M_3}$$

Assignment

```
def interp(expr: Expr, env: Env, mem: Mem): (Value, Mem) = expr match
  ...
  case Assign(name, expr) =>
    val (ev, emem) = interp(expr, env, mem)
    (ev, emem + (lookupId(env, name) -> ev))
```

$$\boxed{\sigma, M \vdash e \Rightarrow v, M}$$

$$\texttt{Assign}\ \frac{\sigma, M \vdash e \Rightarrow v, M' \qquad x \in \mathsf{Domain}(\sigma)}{\sigma, M \vdash x \texttt{ = } e \Rightarrow v, M'[\sigma(x) \mapsto v]}$$

Contents

Call-by-Value vs. Call-by-Reference

The current semantics of MFAE is based on the **call-by-value (CBV)** evaluation strategy, because the argument expression is always evaluated and the result **value** is passed to the parameter.

However, we can define the semantics of MFAE in another way by using the **call-by-reference (CBR)** evaluation strategy instead; if the argument expression is an identifier, the parameter points to its **address**.

CBV

$$\sigma = [\ \mathtt{f} \mapsto \langle \lambda \mathtt{x}.(\ldots), \varnothing \rangle,\ \mathtt{a} \mapsto a_0,\ \ \mathtt{b} \mapsto a_1,\ ]$$

	a_0	a_1	a_2	a_3	a_4
$M =$	1	2			

```
/* MFAE */
var f = x => y => {
  var t = x;
  x = y;
  y = t;
};
var a = 1;
var b = 2;                *
f(a)(b); a; b
```

CBR

$$\sigma = [\ \mathtt{f} \mapsto \langle \lambda \mathtt{x}.(\ldots), \varnothing \rangle,\ \mathtt{a} \mapsto a_0,\ \ \mathtt{b} \mapsto a_1,\ ]$$

	a_0	a_1	a_2
$M =$	1	2	

Call-by-Value vs. Call-by-Reference

PLRG

The current semantics of MFAE is based on the **call-by-value (CBV)** evaluation strategy, because the argument expression is always evaluated and the result **value** is passed to the parameter.

However, we can define the semantics of MFAE in another way by using the **call-by-reference (CBR)** evaluation strategy instead; if the argument expression is an identifier, the parameter points to its **address**.

CBV

$$\sigma = [\ \mathtt{x} \mapsto a_2,\quad \mathtt{y} \mapsto a_3,\ ]$$

	a_0	a_1	a_2	a_3	a_4
$M =$	1	2	1	2	

```
/* MFAE */
var f = x => y => {  *
  var t = x;
  x = y;
  y = t;
};
var a = 1;
var b = 2;
f(a)(b); a; b
```

CBR

$$\sigma = [\ \mathtt{x} \mapsto a_0,\quad \mathtt{y} \mapsto a_1,\ ]$$

	a_0	a_1	a_2
$M =$	1	2	

Call-by-Value vs. Call-by-Reference

PLRG

The current semantics of MFAE is based on the **call-by-value (CBV)** evaluation strategy, because the argument expression is always evaluated and the result **value** is passed to the parameter.

However, we can define the semantics of MFAE in another way by using the **call-by-reference (CBR)** evaluation strategy instead; if the argument expression is an identifier, the parameter points to its **address**.

CBV

$$\sigma = [\ \mathtt{x} \mapsto a_2,\ \ \mathtt{y} \mapsto a_3,\ \ \mathtt{t} \mapsto a_4,\ ]$$

$$M = \begin{array}{ccccc} a_0 & a_1 & a_2 & a_3 & a_4 \\ \hline 1 & 2 & 2 & 1 & 1 \end{array}$$

```
/* MFAE */
var f = x => y => {
  var t = x;
  x = y;
  y = t;                  *
};
var a = 1;
var b = 2;
f(a)(b); a; b
```

CBR

$$\sigma = [\ \mathtt{x} \mapsto a_0,\ \ \mathtt{y} \mapsto a_1,\ \ \mathtt{t} \mapsto a_2,\ ]$$

$$M = \begin{array}{ccc} a_0 & a_1 & a_2 \\ \hline 2 & 1 & 1 \end{array}$$

Call-by-Value vs. Call-by-Reference

The current semantics of MFAE is based on the **call-by-value (CBV)** evaluation strategy, because the argument expression is always evaluated and the result **value** is passed to the parameter.

However, we can define the semantics of MFAE in another way by using the **call-by-reference (CBR)** evaluation strategy instead; if the argument expression is an identifier, the parameter points to its **address**.

CBV

$$\sigma = [\ \mathtt{f} \mapsto \langle \lambda \mathtt{x}.(\ldots), \varnothing \rangle,\ \mathtt{a} \mapsto a_0,\ \ \mathtt{b} \mapsto a_1,\ ]$$

	a_0	a_1	a_2	a_3	a_4
$M =$	1	2	2	1	1

```
/* MFAE */
var f = x => y => {
  var t = x;
  x = y;
  y = t;
};
var a = 1;
var b = 2;
f(a)(b); a; b          *
```

CBR

$$\sigma = [\ \mathtt{f} \mapsto \langle \lambda \mathtt{x}.(\ldots), \varnothing \rangle,\ \mathtt{a} \mapsto a_0,\ \ \mathtt{b} \mapsto a_1,\ ]$$

	a_0	a_1	a_2
$M =$	2	1	1

Function Application (Call-by-Reference)

PLRG

We can define the semantics of MFAE with the **call-by-reference (CBR)** evaluation strategy by adding the following case:

```
def interp(expr: Expr, env: Env, mem: Mem): (Value, Mem) = expr match
  ...
  case App(fun, arg) =>
    val (fv, fmem) = interp(fun, env, mem)
    fv match
      case CloV(param, body, fenv) => arg match
        case Id(name) =>
          val addr = lookupId(env, name)
          interp(body, fenv + (param -> addr), fmem)
        case _ => ...
      case _ => error(s"not a function: ${fv.str}")
    ...
```

$$\texttt{App}_x \; \frac{\sigma, M \vdash e_1 \Rightarrow \langle \lambda x'.e_2, \sigma' \rangle, M_1 \quad x \in \mathsf{Domain}(\sigma) \quad \sigma'[x' \mapsto \sigma(x)], M_1 \vdash e_2 \Rightarrow v_2, M_2}{\sigma, M \vdash e_1(x) \Rightarrow v_2, M_2}$$

Exercise #7

https://github.com/ku-plrg-classroom/docs/tree/main/cose212/mfae

- Please see above document on GitHub:
  - Implement `interp` function.
  - Implement `interpCBR` function.

- It is just an exercise, and you **don't need to submit** anything.
- However, some exam questions might be related to this exercise.

Summary

1. Mutable Variables
2. MFAE – FAE with Mutable Variables
   - Concrete Syntax
   - Abstract Syntax
3. Interpreter and Natural Semantics for MFAE
   - Evaluation with Memories
   - Interpreter and Natural Semantics
   - Mutable Variable
   - Identifier Lookup
   - Function Application
   - Assignment
4. Call-by-Value vs. Call-by-Reference

Next Lecture

- Garbage Collection

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