

# Lecture 3 – Syntax and Semantics (2)

## COSE212: Programming Languages

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2024 Fall

We learned how to define **syntax** and **semantics** of a programming language with (AE) as an example.

- **Syntax**

- Concrete Syntax
- Abstract Syntax
- Concrete vs. Abstract Syntax

- **Semantics**

- Inference Rules
- Big-Step Operational (Natural) Semantics
- Small-Step Operational (Reduction) Semantics

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  - Abstract Syntax
  - Concrete vs. Abstract Syntax
- **Semantics**
  - Inference Rules
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In this lecture, we will learn how to implement the **interpreter** for AE.

- **Parser**: from **strings** to **abstract syntax trees (ASTs)**
- **Interpreter**: from **ASTs** to **values**

## 1. Parsers

- ADTs for Abstract Syntax
- Parsers for Concrete Syntax

## 2. Interpreters

## 1. Parsers

ADTs for Abstract Syntax  
Parsers for Concrete Syntax

## 2. Interpreters

Let's define a Scala **ADT** to represent the **abstract syntax** of AE.

Numbers	$n \in \mathbb{Z}$	(BigInt)
Expressions	$e ::= n$	(Num)
	$e + e$	(Add)
	$e * e$	(Mul)

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// expressions
enum Expr:
  // numbers
  case Num(number: BigInt) // `BigInt` rather than `Int` for integers
  // additions
  case Add(left: Expr, right: Expr)
  // multiplications
  case Mul(left: Expr, right: Expr)
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  // multiplications
  case Mul(left: Expr, right: Expr)
```

For example, an AE expression  $1 + (2 * 3)$  is represented as follows:

```
Add(Num(1), Mul(Num(2), Num(3)))
```



We learned the **concrete syntax** of AE in the last lecture.

```
<expr> ::= <number>
         | <expr> "+" <expr>
         | <expr> "*" <expr>
         | "(" <expr> ")"
```

Then, how can we implement a **parser** for AE?

---

<sup>1</sup><https://github.com/scala/scala-parser-combinators>

<sup>2</sup>[https://en.wikipedia.org/wiki/Parsing\\_expression\\_grammar](https://en.wikipedia.org/wiki/Parsing_expression_grammar)

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```

Then, how can we implement a **parser** for AE?

Let's use **parser combinators** in Scala!

I will explain basic ideas of parser combinators in this lecture. If you are interested in details, please refer to here<sup>1</sup>, and **parsing expression grammars (PEGs)**.<sup>2</sup>

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- **regular expressions** ("**...**".r) as parsers.

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lazy val parser: Parser[String] = "-?[0-9]+".r // parsing integers
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- **regular expressions** ("`...`".`r`) as parsers.

```
lazy val parser: Parser[String] = "-?[0-9]+".r // parsing integers
```

- **combine** them using sequence (`~`, `<~`, `~>`) and alternative (`|`).

```
lazy val parser1: Parser[X] = ...
lazy val parser2: Parser[Y] = ...
lazy val parser3: Parser[X] = ...
parser1 ~ parser2    // Parser[X ~ Y]
parser1 <~ parser2   // Parser[X]   (discard the result of `parser2`)
parser1 ~> parser2   // Parser[Y]   (discard the result of `parser1`)
parser1 | parser3    // Parser[X]
```

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parser1 ~ parser2    // Parser[X ~ Y]
parser1 <~ parser2   // Parser[X]   (discard the result of `parser2`)
parser1 ~> parser2  // Parser[Y]   (discard the result of `parser1`)
parser1 | parser3   // Parser[X]
```

- **transform** the result of a parser using the operator (`^^`).

```
lazy val parser1: Parser[X] = ...
val f: X => Y = ...
parser1 ^^ f // Parser[Y] (apply `f` to the result of `parser1`)
```

For example, let's implement a parser for **list of integers**:

"[]"    "[7]"    "[-042, 4, 20]"

```
type P[+T] = PackratParser[T]
lazy val num : P[BigInt] = "-?[0-9]+".r ^^ { BigInt(_) }
```

For example, let's implement a parser for **list of integers**:

"[]"    "[7]"    "[-042, 4, 20]"

```
type P[+T] = PackratParser[T]
lazy val num  : P[BigInt]      = "-?[0-9]+".r ^^ { BigInt(_) }
lazy val numSeq: P[List[BigInt]] =
  (num <~ ",") ~ numSeq ^^ { case x ~ xs => x :: xs } |
  num           ^^ { case x      => List(x) } |
  ""           ^^ { case _      => Nil      }
```



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  num ^^ { case x => List(x) } |
  "" ^^ { case _ => Nil }
lazy val list : P[List[BigInt]] = "[" ~> numSeq <~ "]"
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  (num <~ ",") ~ numSeq ^^ { case x ~ xs => x :: xs } |
  num ^^ { case x => List(x) } |
  "" ^^ { case _ => Nil }
lazy val list : P[List[BigInt]] = "[" ~> numSeq <~ "]"

parseAll(list, "[]").get // Nil : List[BigInt]
parseAll(list, "[7]").get // List(7) : List[BigInt]
parseAll(list, "[-042, 4, 20]").get // List(-42, 4, 20) : List[BigInt]
```

For example, let's implement a parser for **list of integers**:

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```
type P[+T] = PackratParser[T]
lazy val num  : P[BigInt]      = "-?[0-9]+".r ^^ { BigInt(_) }
lazy val numSeq: P[List[BigInt]] = rep1sep(num, ",")
lazy val list  : P[List[BigInt]] = "[" ~> numSeq <~ "]"

parseAll(list, "[]").get           // Nil           : List[BigInt]
parseAll(list, "[7]").get         // List(7)    : List[BigInt]
parseAll(list, "[-042, 4, 20]").get // List(-42, 4, 20) : List[BigInt]
```

We can simplify it using `rep1sep` (repeat one or more times separated by `","`). There are other helper functions that help us write parsers.

Let's implement a **parser** for AE using Scala **parser combinators**.

```
<expr> ::= <number>  
         | <expr> "+" <expr>  
         | <expr> "*" <expr>  
         | "(" <expr> ")"
```

Let's implement a **parser** for AE using Scala **parser combinators**.

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<expr> ::= <number>
         | <expr> "+" <expr>
         | <expr> "*" <expr>
         | "(" <expr> ")"
```

```
lazy val num: P[BigInt] = "-?[0-9]+".r ^^ { BigInt(_) }
lazy val expr: P[Expr] =
  import Expr.*
  lazy val e0: P[Expr] = (e0<~"+")~e1 ^^ { case l~r => Add(l, r) } | e1
  lazy val e1: P[Expr] = (e1<~"*")~e2 ^^ { case l~r => Mul(l, r) } | e2
  lazy val e2: P[Expr] = num ^^ { case n => Num(n) } | e3
  lazy val e3: P[Expr] = "(" ~> e0 <~ ")"
  e0

parseAll(expr, "42").get // Num(42) : Expr
parseAll(expr, "-1 + 7").get // Add(Num(-1),Num(7)) : Expr
parseAll(expr, "1 + 2 * 3").get // Add(Num(1),Mul(Num(2),Num(3))) : Expr
```

You **don't need to know** the details of parser combinators.

We **provide all parsers** of programming languages in this course.

If you want to use the parser, please just call `Expr` as follows:

```
val x: Expr = Expr("42")           // Num(42)           : Expr
val y: Expr = Expr("-1 + 7")      // Add(Num(-1), Num(7)) : Expr
val z: Expr = Expr("1 + 2 * 3")  // Add(Num(1), Mul(Num(2), Num(3))) : Expr
```

If you want to get the **string form** of the expression, please use `str` method as follows:

```
x.str // "42"           : String
y.str // "(-1 + 7)"    : String
z.str // "(1 + (2 * 3))" : String
```

## 1. Parsers

ADTs for Abstract Syntax

Parsers for Concrete Syntax

## 2. Interpreters

We will implement the **interpreter** for AE according to the following **big-step operational (natural) semantics**:

$$\boxed{\vdash e \Rightarrow n}$$

$$\text{NUM} \frac{}{\vdash n \Rightarrow n}$$

$$\text{ADD} \frac{\vdash e_1 \Rightarrow n_1 \quad \vdash e_2 \Rightarrow n_2}{\vdash e_1 + e_2 \Rightarrow n_1 + n_2}$$

$$\text{MUL} \frac{\vdash e_1 \Rightarrow n_1 \quad \vdash e_2 \Rightarrow n_2}{\vdash e_1 * e_2 \Rightarrow n_1 \times n_2}$$



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```
type Value = BigInt
def interp(expr: Expr): Value = ???
```

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```

type Value = BigInt
def interp(expr: Expr): Value = expr match
  case Num(n)      => ???
  case Add(l, r)   => ???
  case Mul(l, r)   => ???

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```

type Value = BigInt
def interp(expr: Expr): Value = expr match
  case Num(n)    => n
  case Add(l, r) => interp(l) + interp(r)
  case Mul(l, r) => ???

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type Value = BigInt
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```

```
interp(Expr("42"))      // interp(Num(42))                = 42
interp(Expr("-1+7"))    // interp(Add(Num(-1), Num(7)))        = 6
interp(Expr("1+2*3"))  // interp(Add(Num(1), Mul(Num(2), Num(3)))) = 7
```

<https://github.com/ku-plrg-classroom/docs/tree/main/cose212/ae>

- Please see above document on GitHub:
  - Implement `interp` function.
  - Implement `countNums` function.
- It is just an exercise, and you **don't need to submit** anything.
- However, some exam questions might be related to this exercise.

## 1. Parsers

ADTs for Abstract Syntax  
Parsers for Concrete Syntax

## 2. Interpreters



- Identifiers (1)

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