

# Lecture 11 – Mutable Variables

## COSE212: Programming Languages

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2025 Fall

- Mutation makes it possible to **change the state** of a program by **updating the contents** of a data structure or a variable.
  - **Mutable data structures**
  - **Mutable variables**
- **Mutable Data Structures** – Mutable Boxes
- BFAE – FAE with Mutable Boxes
  - Evaluation with Memories
- In this lecture, we will learn **Mutable Variables**
- **MFAE – FAE with Mutable Variables**
  - Concrete and Abstract Syntax
  - Interpreter and Natural Semantics

1. Mutable Variables
2. MFAE – FAE with Mutable Variables
  - Concrete Syntax
  - Abstract Syntax
3. Interpreter and Natural Semantics for MFAE
  - Evaluation with Memories
  - Interpreter and Natural Semantics
  - Mutable Variable
  - Identifier Lookup
  - Function Application
  - Assignment
4. Call-by-Value vs. Call-by-Reference

## 1. Mutable Variables

## 2. MFAE – FAE with Mutable Variables

Concrete Syntax

Abstract Syntax

## 3. Interpreter and Natural Semantics for MFAE

Evaluation with Memories

Interpreter and Natural Semantics

Mutable Variable

Identifier Lookup

Function Application

Assignment

## 4. Call-by-Value vs. Call-by-Reference

A **mutable variable** is a variable whose value can be changed after its initialization.

Let's define mutable variables in Scala:

```
// A mutable variable `x` of type `Int` with 1
var x: Int = 1
x + 2          // 1 + 2 == 3 : Int

// We can reassign a mutable variable `x`
x = 2          // x == 2
x + 2          // 2 + 2 == 4 : Int

// The function `f` is impure because it uses a mutable variable `y`
var y: Int = 1
def f(x: Int): Int = x + y
f(5)           // 5 + 1 == 6 : Int
y = 3
f(5)           // 5 + 3 == 8 : Int
```

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Now, let's extend FAE into MFAE to support **mutable variables**.

```
/* MFAE */  
var x = 5;  
x;           // 5  
x = 8;  
x           // 8
```

```
/* MFAE */  
var y = 1;  
var f = x => { x = x + y; x * x };  
f(5);        // (5 + 1) * (5 + 1) = 36  
y = 3;  
f(5)         // (5 + 3) * (5 + 3) = 64
```

For MFAE, we need to extend **expressions** of FAE with

- 1 **mutable variables** (`var`) rather than immutable variables (`val`)  
(all variables, including parameters, are mutable in MFAE)
- 2 **assignment** (`=`)  
(right-associative: e.g.,  $x = y = e$  is equivalent to  $x = (y = e)$ )
- 3 **sequence** of expressions

```
// expressions
<expr> ::= ...
        | "var" <id> "=" <expr> ";" <expr>
        | <id> "=" <expr>
        | <expr> ";" <expr>
```

For MFAE, we need to extend **expressions** of FAE with

- ① **mutable variables** (`var`) rather than immutable variables (`val`)  
(all variables, including parameters, are mutable in MFAE)
- ② **assignment** (`=`)  
(right-associative: e.g.,  $x = y = e$  is equivalent to  $x = (y = e)$ )
- ③ **sequence** of expressions



Let's define the **abstract syntax** of MFAE in BNF:

Expressions  $\mathbb{E} \ni e ::= \dots$

<code>var x = e; e</code>	(Var)
<code>x = e</code>	(Assign)
<code>e; e</code>	(Seq)

```
enum Expr:
    ...
    // mutable variable definition
    case Var(name: String, init: Expr, body: Expr)
    // variable assignment
    case Assign(name: String, expr: Expr)
    // sequence
    case Seq(left: Expr, right: Expr)
```

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Evaluation with Memories

Interpreter and Natural Semantics

Mutable Variable

Identifier Lookup

Function Application

Assignment

## 4. Call-by-Value vs. Call-by-Reference

We can represent mutable variables by assigning different **addresses** to each variable in the environment and storing their values in the **memory**.

Let's see how to evaluate the following MFAE expression:

```
/* MFAE */  
var x = 5;  
x;  
x = 8;  
x
```

\*

$$\sigma = [$$
  
$$]$$
$$\begin{array}{lcl} \mathbb{A} & : & a_0 \quad a_1 \quad a_2 \quad a_3 \quad \dots \\ M & = & \boxed{\phantom{0}} \boxed{\phantom{0}} \boxed{\phantom{0}} \boxed{\phantom{0}} \boxed{\dots} \end{array}$$

We can represent mutable variables by assigning different **addresses** to each variable in the environment and storing their values in the **memory**.

Let's see how to evaluate the following MFAE expression:

```
/* MFAE */  
var x = 5; *  
x;  
x = 8;  
x
```

$$\sigma = [$$

$x \mapsto a_0$

$$]$$

$\mathbb{A}$	:	$a_0$	$a_1$	$a_2$	$a_3$	$\dots$
$M$	=	5				$\dots$

We can represent mutable variables by assigning different **addresses** to each variable in the environment and storing their values in the **memory**.

Let's see how to evaluate the following MFAE expression:

```
/* MFAE */
var x = 5;
x;           /* 5 */
x = 8;
x
```

$$\sigma = [ \\ \mathbf{x} \mapsto a_0 \\ ]$$

$$\begin{array}{lcl} \mathbb{A} & : & a_0 \quad a_1 \quad a_2 \quad a_3 \quad \dots \\ M & = & \begin{array}{|c|c|c|c|c|} \hline 5 & & & & \dots \\ \hline \end{array} \end{array}$$

We can represent mutable variables by assigning different **addresses** to each variable in the environment and storing their values in the **memory**.

Let's see how to evaluate the following MFAE expression:

```
/* MFAE */  
var x = 5;  
x;           /* 5 */  
x = 8;       *  
x
```

$$\sigma = \left[ \begin{array}{l} \mathbf{x} \mapsto a_0 \end{array} \right]$$

$$\begin{array}{lcl} \mathbb{A} & : & a_0 \quad a_1 \quad a_2 \quad a_3 \quad \dots \\ M & = & \begin{array}{|c|c|c|c|c|} \hline 8 & & & & \dots \\ \hline \end{array} \end{array}$$

We can represent mutable variables by assigning different **addresses** to each variable in the environment and storing their values in the **memory**.

Let's see how to evaluate the following MFAE expression:

```
/* MFAE */
var x = 5;
x;           /* 5 */
x = 8;
x            /* 8 */
```

$$\sigma = [ \\ \mathbf{x} \mapsto a_0 \\ ]$$

$$\begin{array}{l} \mathbb{A} \\ M \end{array} \begin{array}{l} : \\ = \end{array} \begin{array}{cccccc} a_0 & a_1 & a_2 & a_3 & \dots \\ \boxed{8} & \boxed{\phantom{00}} & \boxed{\phantom{00}} & \boxed{\phantom{00}} & \boxed{\dots} \end{array}$$

Here is another MFAE expression:

```
/* MFAE */  
var y = 1;  
var f = x => {  
  x = x + y;  
  x * x  
};  
f(5);  
y = 3;  
f(5)
```

\*

$\sigma = [$

$]$

$\mathbb{A} : a_0 \quad a_1 \quad a_2 \quad a_3 \quad \dots$

$M =$ 

				...
--	--	--	--	-----



Here is another MFAE expression:

```
/* MFAE */
var y = 1;
var f = x => {
  x = x + y;
  x * x
};
f(5);
y = 3;
f(5)
```

\*

$$\sigma = [$$

$$y \mapsto a_0$$

$$]$$

$$\mathbb{A} : a_0 \quad a_1 \quad a_2 \quad a_3 \quad \dots$$

$$M = \begin{array}{|c|c|c|c|c|} \hline 1 & & & & \dots \\ \hline \end{array}$$

Here is another MFAE expression:

```
/* MFAE */
var y = 1;
var f = x => {
  x = x + y;
  x * x
};
f(5);
y = 3;
f(5)
```

\*

$$\sigma = [$$

$$y \mapsto a_0$$

$$f \mapsto a_1$$

$$]$$

$$\mathbb{A} : a_0 \quad a_1 \quad a_2 \quad a_3 \quad \dots$$

$$M = \begin{array}{|c|c|c|c|c|} \hline 1 & v & & & \dots \\ \hline \end{array}$$

where  $v = \langle \lambda x. (x = x + y; x * x), [y \mapsto a_0] \rangle$

Here is another MFAE expression:

```
/* MFAE */
var y = 1;
var f = x => {
  x = x + y;
  x * x
};
f(5);
y = 3;
f(5)
```

\*

$$\sigma = [$$

$$y \mapsto a_0$$

$$x \mapsto a_2$$

$$]$$

$$\mathbb{A} : a_0 \quad a_1 \quad a_2 \quad a_3 \quad \dots$$

$$M = \begin{array}{|c|c|c|c|c|} \hline 1 & v & 5 & & \dots \\ \hline \end{array}$$

where  $v = \langle \lambda x. (x = x + y; x * x), [y \mapsto a_0] \rangle$

Here is another MFAE expression:

```
/* MFAE */
var y = 1;
var f = x => {
  x = x + y; /* 5 + 1 */ *
  x * x
};
f(5);
y = 3;
f(5)
```

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$$]$$

$$\mathbb{A} : a_0 \quad a_1 \quad a_2 \quad a_3 \quad \dots$$

$$M = \begin{array}{|c|c|c|c|c|} \hline 1 & v & 6 & & \dots \\ \hline \end{array}$$

where  $v = \langle \lambda x. (x = x + y; x * x), [y \mapsto a_0] \rangle$

Here is another MFAE expression:

```
/* MFAE */
var y = 1;
var f = x => {
  x = x + y; /* 5 + 1 */
  x * x      /* 6 * 6 */ *
};
f(5);
y = 3;
f(5)
```

$$\sigma = [$$

$$y \mapsto a_0$$

$$x \mapsto a_2$$

$$]$$

$$\mathbb{A} : a_0 \quad a_1 \quad a_2 \quad a_3 \quad \dots$$

$$M = \begin{array}{|c|c|c|c|c|} \hline 1 & v & 6 & & \dots \\ \hline \end{array}$$

where  $v = \langle \lambda x. (x = x + y; x * x), [y \mapsto a_0] \rangle$

Here is another MFAE expression:

```
/* MFAE */
var y = 1;
var f = x => {
  x = x + y;
  x * x
};
f(5);           /* 36 */
y = 3;
f(5)
```

$$\sigma = [$$

$$y \mapsto a_0$$

$$f \mapsto a_1$$

$$]$$

$$\mathbb{A} : a_0 \quad a_1 \quad a_2 \quad a_3 \quad \dots$$

$$M = \begin{array}{|c|c|c|c|c|} \hline 1 & v & 6 & & \dots \\ \hline \end{array}$$

where  $v = \langle \lambda x. (x = x + y; x * x), [y \mapsto a_0] \rangle$

Here is another MFAE expression:

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var y = 1;
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  x = x + y;
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f(5);           /* 36 */
y = 3;          *
f(5)
```

$$\sigma = [$$

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/* MFAE */
var y = 1;
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    x = x + y;
    x * x
};
f(5);          /* 36 */
y = 3;
f(5)
```

$$\sigma = [$$

$$y \mapsto a_0$$

$$x \mapsto a_3$$

$$]$$

$$\mathbb{A} : \begin{matrix} a_0 & a_1 & a_2 & a_3 & \dots \end{matrix}$$

$$M = \begin{matrix} \begin{matrix} 3 & v & 6 & 5 & \dots \end{matrix} \end{matrix}$$

where  $v = \langle \lambda x. (x = x + y; x * x), [y \mapsto a_0] \rangle$



Here is another MFAE expression:

```
/* MFAE */
var y = 1;
var f = x => {
  x = x + y; /* 5 + 3 */ *
  x * x
};
f(5);      /* 36 */
y = 3;
f(5)
```

$$\sigma = [$$

$$y \mapsto a_0$$

$$x \mapsto a_3$$

$$]$$

$$\mathbb{A} : \begin{matrix} a_0 & a_1 & a_2 & a_3 & \dots \end{matrix}$$

$$M = \begin{matrix} \begin{matrix} 3 & v & 6 & 8 & \dots \end{matrix} \end{matrix}$$

where  $v = \langle \lambda x. (x = x + y; x * x), [y \mapsto a_0] \rangle$

Here is another MFAE expression:

```
/* MFAE */
var y = 1;
var f = x => {
  x = x + y; /* 5 + 3 */
  x * x      /* 8 * 8 */ *
};
f(5);        /* 36 */
y = 3;
f(5)
```

$$\sigma = [$$

$$y \mapsto a_0$$

$$x \mapsto a_3$$

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$$\mathbb{A} : a_0 \quad a_1 \quad a_2 \quad a_3 \quad \dots$$

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```

$$\sigma = \left[ \begin{array}{l} y \mapsto a_0 \\ f \mapsto a_1 \end{array} \right]$$

$$\begin{array}{lcl} \mathbb{A} & : & a_0 \quad a_1 \quad a_2 \quad a_3 \quad \dots \\ M & = & \begin{array}{|c|c|c|c|c|} \hline 3 & v & 6 & 8 & \dots \\ \hline \end{array} \end{array}$$

where  $v = \langle \lambda x. (x = x + y; x * x), [y \mapsto a_0] \rangle$

For MFAE, we need to 1) implement the **interpreter** with environments and **memories** by passing the updated memory in the result:

```
def interp(expr: Expr, env: Env, mem: Mem): (Value, Mem) = ???
```

```
type Env = Map[String, Addr]
type Addr = Int
type Mem = Map[Addr, Value]
```

```
enum Value:
  case NumV(n: BigInt)
  case CloV(p: String, b: Expr, e: Env)
```

and 2) define the **natural semantics** with environments and **memories** by passing the updated memory in the result:

$$\sigma, M \vdash e \Rightarrow v, M$$

Environments	$\sigma \in \mathbb{X} \xrightarrow{\text{fin}} \mathbb{A}$	(Env)
Addresses	$a \in \mathbb{A}$	(Addr)
Memories	$M \in \mathbb{A} \xrightarrow{\text{fin}} \mathbb{V}$	(Mem)

```
def interp(expr: Expr, env: Env, mem: Mem): (Value, Mem) = expr match
  ...
  case Var(name, init, body) =>
    val (iv, imem) = interp(init, env, mem)
    val addr = malloc(imem)
    interp(body, env + (name -> addr), imem + (addr -> iv))
```

$$\sigma, M \vdash e \Rightarrow v, M$$

$$\text{Var} \frac{a \notin \text{Domain}(M_1) \quad \sigma, M \vdash e_1 \Rightarrow v_1, M_1 \quad \sigma[x \mapsto a], M_1[a \mapsto v_1] \vdash e_2 \Rightarrow v_2, M_2}{\sigma, M \vdash \text{var } x = e_1; e_2 \Rightarrow v_2, M_2}$$

We learned one way to implement malloc in the previous lecture:

```
def malloc(mem: Mem): Addr = mem.keySet.maxOption.fold(0)(_ + 1)
```

```
def interp(expr: Expr, env: Env, mem: Mem): (Value, Mem) = expr match
  ...
  case Id(name) => (mem(lookupId(env, name)), mem)

def lookupId(env: Env, name: String): Addr =
  env.getOrElse(name, error(s"free identifier: $name"))
```

$$\sigma, M \vdash e \Rightarrow v, M$$

$$\text{Id} \frac{x \in \text{Domain}(\sigma)}{\sigma, M \vdash x \Rightarrow M(\sigma(x)), M}$$

```
def interp(expr: Expr, env: Env, mem: Mem): (Value, Mem) = expr match
...
case App(fun, arg) =>
  val (fv, fmem) = interp(fun, env, mem)
  fv match
    case CloV(param, body, fenv) =>
      val (av, amem) = interp(arg, env, fmem)
      val addr = malloc(amem)
      interp(body, fenv + (param -> addr), amem + (addr -> av))
    case _ =>
      error(s"not a function: ${fv.str}")
```

$$\boxed{\sigma, M \vdash e \Rightarrow v, M}$$

$$\text{App} \frac{\sigma, M \vdash e_1 \Rightarrow \langle \lambda x. e_3, \sigma' \rangle, M_1 \quad \sigma, M_1 \vdash e_2 \Rightarrow v_2, M_2 \quad a \notin \text{Domain}(M_2) \quad \sigma'[x \mapsto a], M_2[a \mapsto v_2] \vdash e_3 \Rightarrow v_3, M_3}{\sigma, M \vdash e_1(e_2) \Rightarrow v_3, M_3}$$

```
def interp(expr: Expr, env: Env, mem: Mem): (Value, Mem) = expr match
...
case Assign(name, expr) =>
  val (ev, emem) = interp(expr, env, mem)
  (ev, emem + (lookupId(env, name) -> ev))
```

$$\boxed{\sigma, M \vdash e \Rightarrow v, M}$$

$$\text{Assign} \frac{\sigma, M \vdash e \Rightarrow v, M' \quad x \in \text{Domain}(\sigma)}{\sigma, M \vdash x = e \Rightarrow v, M'[\sigma(x) \mapsto v]}$$



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4. Call-by-Value vs. Call-by-Reference

The current semantics of MFAE is based on the **call-by-value (CBV)** evaluation strategy, because the argument expression is always evaluated and the result **value** is passed to the parameter.

However, we can define the semantics of MFAE in another way by using the **call-by-reference (CBR)** evaluation strategy instead; if the argument expression is an identifier, the parameter points to its **address**.

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## CBV

$$\sigma = [$$

$$f \mapsto a_0,$$

$$a \mapsto a_1,$$

$$b \mapsto a_2,$$

$$]$$

$a_0$	$a_1$	$a_2$	$a_3$	$a_4$	$a_5$
$\langle \lambda \dots \rangle$	1	2			

/\* MFAE \*/

```
var f = x => y => {
  var t = x;
  x = y;
  y = t;
};
var a = 1;
var b = 2;
f(a)(b); a; b
```

\*

## CBR

$$\sigma = [$$

$$f \mapsto a_0,$$

$$a \mapsto a_1,$$

$$b \mapsto a_2,$$

$$]$$

$a_0$	$a_1$	$a_2$	$a_3$
$\langle \lambda \dots \rangle$	1	2	

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**CBV**

$$\sigma = [$$

$$x \mapsto a_3,$$

$$y \mapsto a_4,$$

$$]$$

$a_0$	$a_1$	$a_2$	$a_3$	$a_4$	$a_5$
$\langle \lambda \dots \rangle$	1	2	1	2	

*/\* MFAE \*/*

```
var f = x => y => { *
  var t = x;
  x = y;
  y = t;
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var a = 1;
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f(a)(b); a; b
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**CBR**

$$\sigma = [$$

$$x \mapsto a_1,$$

$$y \mapsto a_2,$$

$$]$$

$a_0$	$a_1$	$a_2$	$a_3$
$\langle \lambda \dots \rangle$	1	2	

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## CBV

$$\sigma = [$$

$$x \mapsto a_3,$$

$$y \mapsto a_4,$$

$$t \mapsto a_5,$$

$$]$$

$a_0$	$a_1$	$a_2$	$a_3$	$a_4$	$a_5$
$\langle \lambda \dots \rangle$	1	2	1	2	1

/\* MFAE \*/

```
var f = x => y => {
  var t = x;
  x = y;
  y = t;
};
var a = 1;
var b = 2;
f(a)(b); a; b
```

\*

## CBR

$$\sigma = [$$

$$x \mapsto a_1,$$

$$y \mapsto a_2,$$

$$t \mapsto a_3,$$

$$]$$

$a_0$	$a_1$	$a_2$	$a_3$
$\langle \lambda \dots \rangle$	1	2	1

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**CBV**

$$\sigma = [$$

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$$]$$

$a_0$	$a_1$	$a_2$	$a_3$	$a_4$	$a_5$
$\langle \lambda \dots \rangle$	1	2	2	2	1

/\* MFAE \*/

```
var f = x => y => {
  var t = x;
  x = y;
  y = t;
};
var a = 1;
var b = 2;
f(a)(b); a; b
```

\*

**CBR**

$$\sigma = [$$

$$x \mapsto a_1,$$

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$$t \mapsto a_3,$$

$$]$$

$a_0$	$a_1$	$a_2$	$a_3$
$\langle \lambda \dots \rangle$	2	2	1

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**CBV**

$$\sigma = [$$

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$$t \mapsto a_5,$$

$$]$$

$a_0$	$a_1$	$a_2$	$a_3$	$a_4$	$a_5$
$\langle \lambda \dots \rangle$	1	2	2	1	1

*/\* MFAE \*/*

```
var f = x => y => {
  var t = x;
  x = y;
  y = t;
};
var a = 1;
var b = 2;
f(a)(b); a; b
```

\*

**CBR**

$$\sigma = [$$

$$x \mapsto a_1,$$

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$$t \mapsto a_3,$$

$$]$$

$a_0$	$a_1$	$a_2$	$a_3$
$\langle \lambda \dots \rangle$	2	1	1

The current semantics of MFAE is based on the **call-by-value (CBV)** evaluation strategy, because the argument expression is always evaluated and the result **value** is passed to the parameter.

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$$b \mapsto a_2,$$

$$]$$

$a_0$	$a_1$	$a_2$	$a_3$	$a_4$	$a_5$
$\langle \lambda \dots \rangle$	1	2	2	1	1

/\* MFAE \*/

```
var f = x => y => {
  var t = x;
  x = y;
  y = t;
};
var a = 1;
var b = 2;
f(a)(b); a; b
```

\*

**CBR**

$$\sigma = [$$

$$f \mapsto a_0,$$

$$a \mapsto a_1,$$

$$b \mapsto a_2,$$

$$]$$

$a_0$	$a_1$	$a_2$	$a_3$
$\langle \lambda \dots \rangle$	2	1	1



We can define the semantics of MFAE with the **call-by-reference (CBR)** evaluation strategy by adding the following case:

```
def interp(expr: Expr, env: Env, mem: Mem): (Value, Mem) = expr match
...
case App(fun, arg) =>
  val (fv, fmem) = interp(fun, env, mem)
  fv match
    case CloV(param, body, fenv) => arg match
      case Id(name) =>
        val addr = lookupId(env, name)
        interp(body, fenv + (param -> addr), fmem)
      case _ => ...
    case _ => error(s"not a function: ${fv.str}")
...

```

$$\text{App}_x \frac{\sigma, M \vdash e_1 \Rightarrow \langle \lambda x'. e_2, \sigma' \rangle, M_1 \quad x \in \text{Domain}(\sigma) \quad \sigma'[x' \mapsto \sigma(x)], M_1 \vdash e_2 \Rightarrow v_2, M_2}{\sigma, M \vdash e_1(x) \Rightarrow v_2, M_2}$$

<https://github.com/ku-plrg-classroom/docs/tree/main/cose212/mfae>

- Please see above document on GitHub:
  - Implement `interp` function.
  - Implement `interpCBR` function.
- It is just an exercise, and you **don't need to submit** anything.
- However, some exam questions might be related to this exercise.

1. Mutable Variables
2. MFAE – FAE with Mutable Variables
  - Concrete Syntax
  - Abstract Syntax
3. Interpreter and Natural Semantics for MFAE
  - Evaluation with Memories
  - Interpreter and Natural Semantics
  - Mutable Variable
  - Identifier Lookup
  - Function Application
  - Assignment
4. Call-by-Value vs. Call-by-Reference

- Garbage Collection

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