

Lecture 10 – Mutable Data Structures

COSE212: Programming Languages

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2026 Fall

- Recursion
 - Recursion in F1VAE and FVAE
 - `mkRec` helper function
 - RFAE – FAE with recursion and conditionals
- In this lecture, we will learn **mutable data structures** (boxes)
- BFAE – FAE **with mutable boxes**
 - Concrete and Abstract Syntax
 - Interpreter and Natural Semantics

1. Mutable Data Structures

2. BFAE – FAE with Mutable Boxes

- Concrete Syntax

- Abstract Syntax

3. Interpreter and Natural Semantics for BFAE

- Evaluation with Memories

- Interpreter and Natural Semantics

- Addition

- Box Creation

- Box Content Getter

- Box Content Setter

- Sequence

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So far, our languages are **purely functional**:

- All functions are **pure** (no side effects)
- All data structures and variables are **immutable** (no mutation)

However, **mutation** is widely used in practice, especially in **imperative languages** (e.g., C, C++, Java, Python, etc.).

Mutation makes it possible to **change the state** of a program by **updating the contents** of a data structure or a variable after its creation.

- **Mutable data structures** (e.g., `mutable.Map` in Scala)
- **Mutable variables** (e.g., `var` in Scala)

While mutation helps us write more **efficient** programs, it also makes programs **harder to reason** about and **error-prone**.

In this lecture, we will learn **mutable data structures**.

A **mutable data structure** is a data structure whose **contents** can be **modified** after its creation. Let's define them in Scala:

```
// immutable map
val imap = Map("x" -> 1, "y" -> 2)
imap + ("x" -> 3)    // Map(x -> 3, y -> 2)
imap                // Map(x -> 1, y -> 2)

// mutable map
val mmap = scala.collection.mutable.Map("x" -> 1, "y" -> 2)
mmap.update("x", 3)
mmap                // mutable.Map(x -> 3, y -> 2)
```

We can define our own **mutable data structure** – a **Box**:

```
// mutable box
case class Box(var content: Int)
val box: Box = Box(5)
box.content    // 5
box.content = 8
box.content    // 8
```

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Now, let's extend FAE into BFAE to support **mutable boxes**.

```
/* BFAE */  
val box = Box(5);  
box.get;           // 5  
box.set(8);  
box.get            // 8
```

```
/* BFAE */  
val box = Box(1);  
val f    = x => x + box.get;  
f(3);    // 3 + 1 = 4  
box.set(2);  
f(3)     // 3 + 2 = 5
```

(We support variable definitions (`val`) as syntactic sugar.)

For BFAE, we need to extend **expressions** of FAE with

- 1 **box creation** (`Box`)
- 2 **box operations**: content getter (`get`) and setter (`set`)
- 3 **sequence** of expressions


```
// expressions
<expr> ::= ...
        | "Box" "(" <expr> ")"
        | <expr> "." "get"
        | <expr> "." "set" "(" <expr> ")"
        | <expr> ";" <expr>
```

For BFAE, we need to extend **expressions** of FAE with

- ① **box creation**
- ② **box operations:** get and set
- ③ **sequence** of expressions

Let's define the **abstract syntax** of BFAE in BNF:

Expressions $\mathbb{E} \ni e ::= \dots$

	Box(<i>e</i>)	(NewBox)
	<i>e</i> .get	(GetBox)
	<i>e</i> .set(<i>e</i>)	(SetBox)
	<i>e</i> ; <i>e</i>	(Seq)

```
enum Expr:
```

```
...
```

```
// box creation
```

```
case NewBox(expr: Expr)
```

```
// box content getter
```

```
case GetBox(box: Expr)
```

```
// box content setter
```

```
case SetBox(box: Expr, expr: Expr)
```

```
// sequence
```

```
case Seq(left: Expr, right: Expr)
```

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How to evaluate the following BFAE expression?

```
/* BFAE */  
val box = Box(5);  
box.get;    // 5  
box.set(8);  
box.get     // 8
```

Let's evaluate it with a **memory** M , which is a finite **mapping** from **addresses** to their **values**.

$$M \in \mathbb{A} \xrightarrow{\text{fin}} \mathbb{V}$$

A **box** allocates a **memory cell** to store a **value** in the **memory**.

- **box creation** allocates a memory cell and stores the value
- **box content getter** reads the value from the memory cell
- **box content setter** writes the value to the memory cell

How to evaluate the following BFAE expression?

```
/* BFAE */
```

```
val box = Box(5);
```

```
box.get;
```

```
box.set(8);
```

```
box.get
```

*

$$\sigma = [$$

$$\begin{array}{lcl} \mathbb{A} & : & a_0 \quad a_1 \quad a_2 \quad \dots \\ M & = & \begin{array}{|c|c|c|c|} \hline & & & \dots \\ \hline \end{array} \end{array}$$

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How to evaluate the following BFAE expression?

```
/* BFAE */
```

```
val box = Box(5); *  
box.get;  
box.set(8);  
box.get
```

$$\sigma = [\quad \quad \quad]$$

$$\text{box} \mapsto a_0$$

$$\begin{array}{lcl} \mathbb{A} & : & a_0 \quad a_1 \quad a_2 \quad \dots \\ M & = & \begin{array}{|c|c|c|c|} \hline 5 & & & \dots \\ \hline \end{array} \end{array}$$

Let's evaluate it with a **memory** M , which is a **mapping** from **addresses** to **values**.

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How to evaluate the following BFAE expression?

```
/* BFAE */
val box = Box(5);
box.get; /* 5 */ *
box.set(8);
box.get
```

$$\sigma = \left[\begin{array}{l} \text{box} \mapsto a_0 \end{array} \right]$$

$$\begin{array}{lcl} \mathbb{A} & : & a_0 \quad a_1 \quad a_2 \quad \dots \\ M & = & \begin{array}{|c|c|c|c|} \hline 5 & & & \dots \\ \hline \end{array} \end{array}$$

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How to evaluate the following BFAE expression?

```
/* BFAE */
```

```
val box = Box(5);
```

```
box.get; /* 5 */
```

```
box.set(8);
```

```
*
```

```
box.get
```

$$\sigma = [$$

$$\text{box} \mapsto a_0$$

$$]$$

$$\mathbb{A} : a_0 \quad a_1 \quad a_2 \quad \dots$$

$$M = \begin{array}{|c|c|c|c|} \hline 8 & & & \dots \\ \hline \end{array}$$

Let's evaluate it with a **memory** M , which is a **mapping** from **addresses** to **values**.

$$M \in \mathbb{A} \xrightarrow{\text{fin}} \mathbb{V}$$

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How to evaluate the following BFAE expression?

```
/* BFAE */
val box = Box(5);
box.get; /* 5 */
box.set(8);
box.get /* 8 */ *
```

$$\sigma = \left[\begin{array}{l} \text{box} \mapsto a_0 \end{array} \right]$$

$$\begin{array}{lcl} \mathbb{A} & : & a_0 \quad a_1 \quad a_2 \quad \dots \\ M & = & \begin{array}{|c|c|c|c|} \hline 8 & & & \dots \\ \hline \end{array} \end{array}$$

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- **box content setter** writes the value to the memory cell

Here is another BFAE expression:

```
/* BFAE */  
val a = Box(1);  
val f = x => x + a.get;  
f(5);  
  
a.set(2);  
f(5);  
  
val b = Box(a);  
b.get.set(3);  
f(5)
```

*

$\sigma = [$

 $]$

$\mathbb{A} : a_0 \quad a_1 \quad a_2 \quad \dots$
 $M =$

			...
--	--	--	-----

Here is another BFAE expression:

```
/* BFAE */  
val a = Box(1);  
val f = x => x + a.get;  
f(5);  
  
a.set(2);  
f(5);  
  
val b = Box(a);  
b.get.set(3);  
f(5)
```

*

$$\sigma = [$$
$$a \mapsto a_0$$
$$]$$

$$\begin{array}{lcl} \mathbb{A} & : & a_0 \quad a_1 \quad a_2 \quad \dots \\ M & = & \begin{array}{|c|c|c|c|} \hline 1 & & & \dots \\ \hline \end{array} \end{array}$$

Here is another BFAE expression:

```
/* BFAE */  
val a = Box(1);  
val f = x => x + a.get; *  
f(5);  
  
a.set(2);  
f(5);  
  
val b = Box(a);  
b.get.set(3);  
f(5)
```

$$\sigma = [$$
$$a \mapsto a_0$$
$$f \mapsto \langle \lambda x. (x + a.get), [a \mapsto a_0] \rangle$$
$$]$$
$$\begin{array}{lcl} \mathbb{A} & : & a_0 \quad a_1 \quad a_2 \quad \dots \\ M & = & \begin{array}{|c|c|c|c|} \hline 1 & & & \dots \\ \hline \end{array} \end{array}$$

Here is another BFAE expression:

```
/* BFAE */  
val a = Box(1);  
val f = x => x + a.get;  
f(5);    /* 5 + 1 = 6 */ *  
  
a.set(2);  
f(5);  
  
val b = Box(a);  
b.get.set(3);  
f(5)
```

$$\sigma = [$$
$$a \mapsto a_0$$
$$f \mapsto \langle \lambda x. (x + a.get), [a \mapsto a_0] \rangle$$
$$]$$

$$\begin{array}{lcl} \mathbb{A} & : & a_0 \quad a_1 \quad a_2 \quad \dots \\ M & = & \begin{array}{|c|c|c|c|} \hline 1 & & & \dots \\ \hline \end{array} \end{array}$$

Here is another BFAE expression:

```
/* BFAE */  
val a = Box(1);  
val f = x => x + a.get;  
f(5);    /* 5 + 1 = 6 */  
  
a.set(2);  
f(5);  
  
val b = Box(a);  
b.get.set(3);  
f(5)
```

*

$$\sigma = [$$
$$a \mapsto a_0$$
$$f \mapsto \langle \lambda x. (x + a.get), [a \mapsto a_0] \rangle$$
$$]$$
$$\begin{array}{lcl} \mathbb{A} & : & a_0 \quad a_1 \quad a_2 \quad \dots \\ M & = & \begin{array}{|c|c|c|c|} \hline 2 & & & \dots \\ \hline \end{array} \end{array}$$

Here is another BFAE expression:

```
/* BFAE */  
val a = Box(1);  
val f = x => x + a.get;  
f(5);    /* 5 + 1 = 6 */  
  
a.set(2);  
f(5);    /* 5 + 2 = 7 */ *  
  
val b = Box(a);  
b.get.set(3);  
f(5)
```

$$\sigma = [$$
$$a \mapsto a_0$$
$$f \mapsto \langle \lambda x. (x + a.get), [a \mapsto a_0] \rangle$$
$$]$$

$$\begin{array}{lcl} \mathbb{A} & : & a_0 \quad a_1 \quad a_2 \quad \dots \\ M & = & \begin{array}{|c|c|c|c|} \hline 2 & & & \dots \\ \hline \end{array} \end{array}$$

Here is another BFAE expression:

```
/* BFAE */  
val a = Box(1);  
val f = x => x + a.get;  
f(5);    /* 5 + 1 = 6 */  
  
a.set(2);  
f(5);    /* 5 + 2 = 7 */  
  
val b = Box(a);  
b.get.set(3);  
f(5)
```

*

$$\sigma = [$$
$$a \mapsto a_0$$
$$f \mapsto \langle \lambda x. (x + a.get), [a \mapsto a_0] \rangle$$
$$b \mapsto a_1$$
$$]$$
$$\begin{array}{lcl} \mathbb{A} & : & a_0 \quad a_1 \quad a_2 \quad \dots \\ M & = & \begin{array}{|c|c|c|c|} \hline 2 & a_0 & & \dots \\ \hline \end{array} \end{array}$$

Here is another BFAE expression:

```
/* BFAE */  
val a = Box(1);  
val f = x => x + a.get;  
f(5);    /* 5 + 1 = 6 */  
  
a.set(2);  
f(5);    /* 5 + 2 = 7 */  
  
val b = Box(a);  
b.get.set(3);  
f(5)
```

*

$$\sigma = [$$
$$a \mapsto a_0$$
$$f \mapsto \langle \lambda x. (x + a.get), [a \mapsto a_0] \rangle$$
$$b \mapsto a_1$$
$$]$$
$$\begin{array}{lcl} \mathbb{A} & : & a_0 \quad a_1 \quad a_2 \quad \dots \\ M & = & \begin{array}{|c|c|c|c|} \hline 3 & a_0 & & \dots \\ \hline \end{array} \end{array}$$

Here is another BFAE expression:

```
/* BFAE */
val a = Box(1);
val f = x => x + a.get;
f(5);    /* 5 + 1 = 6 */

a.set(2);
f(5);    /* 5 + 2 = 7 */

val b = Box(a);
b.get.set(3);
f(5)     /* 5 + 3 = 8 */ *
```

$$\sigma = [$$
$$a \mapsto a_0$$
$$f \mapsto \langle \lambda x. (x + a.get), [a \mapsto a_0] \rangle$$
$$b \mapsto a_1$$
$$]$$
$$\begin{array}{lcl} \mathbb{A} & : & a_0 \quad a_1 \quad a_2 \quad \dots \\ M & = & \begin{array}{|c|c|c|c|} \hline 3 & a_0 & & \dots \\ \hline \end{array} \end{array}$$

For BFAE, we need to 1) implement the **interpreter** with environments and **memories** by passing the updated memory in the result:

```
def interp(expr: Expr, env: Env, mem: Mem): (Value, Mem) = ???
```

```
type Addr = Int
type Mem = Map[Addr, Value]
enum Value:
  ...
  case BoxV(addr: Addr)
```

and 2) define the **natural semantics** with environments and **memories** by passing the updated memory in the result:

$$\sigma, M \vdash e \Rightarrow v, M$$

Addresses $a \in \mathbb{A}$ (Addr)

Memories $M \in \mathbb{A} \xrightarrow{\text{fin}} \mathbb{V}$ (Mem)

Values $\mathbb{V} \ni v ::= \dots \mid a$ (BoxV)

```
def interp(expr: Expr, env: Env, mem: Mem): (Value, Mem) = expr match
  ...
  case Add(l, r) =>
    val (lv, lmem) = interp(l, env, mem)
    val (rv, rmem) = interp(r, env, lmem)
    (numAdd(lv, rv), rmem)
```

$$\sigma, M \vdash e \Rightarrow v, M$$

$$\text{Add} \frac{\sigma, M \vdash e_1 \Rightarrow n_1, M_1 \quad \sigma, M_1 \vdash e_2 \Rightarrow n_2, M_2}{\sigma, M \vdash e_1 + e_2 \Rightarrow n_1 + n_2, M_2}$$

```
/* BFAE */
val x = Box(5);
{ x.set(8); 2 } + x.get // 2 + 8 = 10 -- NOT 2 + 5 = 7
```

```
def interp(expr: Expr, env: Env, mem: Mem): (Value, Mem) = expr match
  ...
  case NewBox(c) =>
    val (cv, cmem) = interp(c, env, mem)
    val addr = malloc(cmem)
    (BoxV(addr), cmem + (addr -> cv))
```

$$\sigma, M \vdash e \Rightarrow v, M$$

$$\text{NewBox} \frac{\sigma, M \vdash e \Rightarrow v, M_1 \quad a \notin \text{dom}(M_1)}{\sigma, M \vdash \text{Box}(e) \Rightarrow a, M_1[a \mapsto v]}$$

One way to implement malloc is to find the maximum address in the memory and increment it by one, 0 if the memory is empty:

```
def malloc(mem: Mem): Addr = mem.keySet.maxOption.fold(0)(_ + 1)
```

```
def interp(expr: Expr, env: Env, mem: Mem): (Value, Mem) = expr match
...
case GetBox(b) =>
  val (bv, bmem) = interp(b, env, mem)
  bv match
    case BoxV(addr) => (bmem(addr), bmem)
    case _          => error(s"not a box: ${bv.str}")
```

$$\boxed{\sigma, M \vdash e \Rightarrow v, M}$$

$$\text{GetBox} \frac{\sigma, M \vdash e \Rightarrow a, M_1}{\sigma, M \vdash e.\text{get} \Rightarrow M_1(a), M_1}$$

```
def interp(expr: Expr, env: Env, mem: Mem): (Value, Mem) = expr match
...
case SetBox(b, c) =>
  val (bv, bmem) = interp(b, env, mem)
  bv match
    case BoxV(addr) =>
      val (cv, cmem) = interp(c, env, bmem)
      (cv, cmem + (addr -> cv))
    case _ =>
      error(s"not a box: ${bv.str}")
```

$$\boxed{\sigma, M \vdash e \Rightarrow v, M}$$

$$\text{SetBox} \frac{\sigma, M \vdash e_1 \Rightarrow a, M_1 \quad \sigma, M_1 \vdash e_2 \Rightarrow v, M_2}{\sigma, M \vdash e_1.\text{set}(e_2) \Rightarrow v, M_2[a \mapsto v]}$$

```
def interp(expr: Expr, env: Env, mem: Mem): (Value, Mem) = expr match
  ...
  case Seq(l, r) =>
    val (_, lmem) = interp(l, env, mem)
    interp(r, env, lmem)
```

$$\boxed{\sigma, M \vdash e \Rightarrow v, M}$$

$$\text{Seq} \frac{\sigma, M \vdash e_1 \Rightarrow _, M_1 \quad \sigma, M_1 \vdash e_2 \Rightarrow v_2, M_2}{\sigma, M \vdash e_1; e_2 \Rightarrow v_2, M_2}$$

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<https://github.com/ku-plrg-classroom/docs/tree/main/cose212/bfae>

- Please see above document on GitHub:
 - Implement `interp` function.
- It is just an exercise, and you **don't need to submit** anything.
- However, some exam questions might be related to this exercise, so we **strongly recommend** solving it by yourself.

- Mutable Variables

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