

Lecture 3 – Syntax and Semantics (2)

COSE212: Programming Languages

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We learned how to define **syntax** and **semantics** of a programming language with (AE) as an example.

- **Syntax**
 - Concrete Syntax
 - Abstract Syntax
 - Concrete vs. Abstract Syntax
- **Semantics**
 - Inference Rules
 - Big-Step Operational (Natural) Semantics
 - Small-Step Operational Semantics

In this lecture, we will learn how to implement the **interpreter** for AE.

- **Parser**: from **strings** to **abstract syntax trees (ASTs)**
- **Interpreter**: from **ASTs** to **values**

<https://github.com/ku-plrg-classroom/docs/tree/main/cose212/ae>

1. Parsers

ADTs for Abstract Syntax

Parsers for Concrete Syntax

2. Interpreters

1. Parsers

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2. Interpreters

Let's define a Scala **ADT** to represent the **abstract syntax** of AE.

Numbers	$n \in \mathbb{Z}$	(BigInt)
Expressions	$e ::= n$	(Num)
	$e + e$	(Add)
	$e * e$	(Mul)

```
// expressions
enum Expr:
  // numbers
  case Num(number: BigInt) // `BigInt` rather than `Int` for integers
  // additions
  case Add(left: Expr, right: Expr)
  // multiplications
  case Mul(left: Expr, right: Expr)
```

For example, an AE expression $1 + (2 * 3)$ is represented as follows:

```
Add(Num(1), Mul(Num(2), Num(3)))
```

We learned the **concrete syntax** of AE in the last lecture.

```
<expr> ::= <number>
         | <expr> "+" <expr>
         | <expr> "*" <expr>
         | "(" <expr> ")"
```

Then, how can we implement a **parser** for AE?

Let's use **parser combinators** in Scala!

I will explain basic ideas of parser combinators in this lecture. If you are interested in details, please refer to here¹, and **parsing expression grammars (PEGs)**.²

¹<https://github.com/scala/scala-parser-combinators>

²https://en.wikipedia.org/wiki/Parsing_expression_grammar

What is a **parser** in Scala parser combinators?

A function that takes a **string** and returns its **result** and **remaining string**.

$$(\text{Input String}) \xRightarrow{[\text{PARSER}]} (\text{Result}, \text{Remaining String})$$

For example, if we have a parser that recognizes an integer at the beginning of a string, it works as follows:

$$\text{"14 is integer"} \xRightarrow{[\text{INTEGER PARSER}]} (14, \text{" is integer"})$$

How can we construct **parsers** in Scala?

- **regular expressions** ("**...**".r) as parsers.

```
lazy val parser: Parser[String] = "-?[0-9]+".r // parsing integers
```

- **combine** them using sequence (~, <~, ~>) and alternative (|).

```
lazy val parser1: Parser[X] = ...
lazy val parser2: Parser[Y] = ...
lazy val parser3: Parser[X] = ...
parser1 ~ parser2    // Parser[X ~ Y]
parser1 <~ parser2    // Parser[X]   (discard the result of `parser2`)
parser1 ~> parser2    // Parser[Y]   (discard the result of `parser1`)
parser1 | parser3     // Parser[X]
```

- **transform** the result of a parser using the operator (^~).

```
lazy val parser1: Parser[X] = ...
val f: X => Y = ...
parser1 ^~ f // Parser[Y]   (apply `f` to the result of `parser1`)
```


For example, let's implement a parser for **list of integers**:

"[]" "[7]" "[-042, 4, 20]"

```
type P[+T] = PackratParser[T]
lazy val num : P[BigInt] = "-?[0-9]+".r ^^ { BigInt(_) }
```

For example, let's implement a parser for **list of integers**:

"[]" "[7]" "[-042, 4, 20]"

```
type P[+T] = PackratParser[T]
lazy val num  : P[BigInt]      = "-?[0-9]+".r ^^ { BigInt(_) }
lazy val numSeq: P[List[BigInt]] =
  (num <~ ",") ~ numSeq ^^ { case x ~ xs => x :: xs } |
  num           ^^ { case x      => List(x) } |
  ""           ^^ { case _      => Nil      }
```

For example, let's implement a parser for **list of integers**:

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```
type P[+T] = PackratParser[T]
lazy val num    : P[BigInt]      = "-?[0-9]+".r ^^ { BigInt(_) }
lazy val numSeq: P[List[BigInt]] =
  (num <~ ",") ~ numSeq ^^ { case x ~ xs => x :: xs } |
  num           ^^ { case x      => List(x) } |
  ""           ^^ { case _      => Nil      }
lazy val list   : P[List[BigInt]] = "[" ~> numSeq <~ "]"
```

For example, let's implement a parser for **list of integers**:

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type P[+T] = PackratParser[T]
lazy val num  : P[BigInt]      = "-?[0-9]+".r ^^ { BigInt(_) }
lazy val numSeq: P[List[BigInt]] =
  (num <~ ",") ~ numSeq ^^ { case x ~ xs => x :: xs } |
  num              ^^ { case x      => List(x) } |
  ""              ^^ { case _      => Nil      }
lazy val list  : P[List[BigInt]] = "[" ~> numSeq <~ "]"

parseAll(list, "[]").get           // Nil           : List[BigInt]
parseAll(list, "[7]").get          // List(7)        : List[BigInt]
parseAll(list, "[-042, 4, 20]").get // List(-42, 4, 20) : List[BigInt]
```

For example, let's implement a parser for **list of integers**:

"[]" "[7]" "[-042, 4, 20]"

```
type P[+T] = PackratParser[T]
lazy val num    : P[BigInt]      = "-?[0-9]+".r ^^ { BigInt(_) }
lazy val numSeq: P[List[BigInt]] = repsep(num, ",")
lazy val list   : P[List[BigInt]] = "[" ~> numSeq <~ "]"

parseAll(list, "[]").get           // Nil           : List[BigInt]
parseAll(list, "[7]").get          // List(7)      : List[BigInt]
parseAll(list, "[-042, 4, 20]").get // List(-42, 4, 20) : List[BigInt]
```

We can simplify it using `repsep` (repeat zero or more times separated by `", "`). There are other helper functions that help us write parsers.

Let's implement a **parser** for AE using Scala **parser combinators**.

```
<expr> ::= <number>
         | <expr> "+" <expr>
         | <expr> "*" <expr>
         | "(" <expr> ")"
```

```
lazy val num: P[BigInt] = "-?[0-9]+".r ^^ { BigInt(_) }
lazy val expr: P[Expr] =
  import Expr.*
  lazy val e0: P[Expr] = (e0<~"+" )~e1 ^^ { case l~r => Add(l, r) } | e1
  lazy val e1: P[Expr] = (e1<~"*" )~e2 ^^ { case l~r => Mul(l, r) } | e2
  lazy val e2: P[Expr] = num ^^ { case n => Num(n) } | e3
  lazy val e3: P[Expr] = "(" ~> e0 <~ ")"
  e0

parseAll(expr, "42").get // Num(42) : Expr
parseAll(expr, "-1 + 7").get // Add(Num(-1),Num(7)) : Expr
parseAll(expr, "1 + 2 * 3").get // Add(Num(1),Mul(Num(2),Num(3))) : Expr
```

You **don't need to know** the details of parser combinators.

We **provide all parsers** of programming languages in this course.

If you want to use the parser, please just call `Expr` as follows:

```
val x: Expr = Expr("42")           // Num(42)           : Expr
val y: Expr = Expr("-1 + 7")       // Add(Num(-1),Num(7)) : Expr
val z: Expr = Expr("1 + 2 * 3")    // Add(Num(1),Mul(Num(2),Num(3))) : Expr
```

If you want to get the **string form** of the expression, please use `str` method as follows:

```
x.str    // "42"           : String
y.str    // "(-1 + 7)"      : String
z.str    // "(1 + (2 * 3))" : String
```

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2. Interpreters

We will implement the **interpreter** for AE according to the following **big-step operational (natural) semantics**:

$$\boxed{\vdash e \Rightarrow n}$$

$$\text{NUM} \frac{}{\vdash n \Rightarrow n} \quad \text{ADD} \frac{\vdash e_1 \Rightarrow n_1 \quad \vdash e_2 \Rightarrow n_2}{\vdash e_1 + e_2 \Rightarrow n_1 + n_2} \quad \text{MUL} \frac{\vdash e_1 \Rightarrow n_1 \quad \vdash e_2 \Rightarrow n_2}{\vdash e_1 * e_2 \Rightarrow n_1 \times n_2}$$

```
type Value = BigInt
def interp(expr: Expr): Value = ???
```

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```
type Value = BigInt
def interp(expr: Expr): Value = expr match
  case Num(n)      => ???
  case Add(l, r)   => ???
  case Mul(l, r)   => ???
```

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```
type Value = BigInt
def interp(expr: Expr): Value = expr match
  case Num(n)      => n
  case Add(l, r)   => ???
  case Mul(l, r)   => ???
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```
type Value = BigInt
def interp(expr: Expr): Value = expr match
  case Num(n)      => n
  case Add(l, r)   => interp(l) + interp(r)
  case Mul(l, r)   => ???
```

We will implement the **interpreter** for AE according to the following **big-step operational (natural) semantics**:

$$\boxed{\vdash e \Rightarrow n}$$

$$\text{NUM} \frac{}{\vdash n \Rightarrow n} \quad \text{ADD} \frac{\vdash e_1 \Rightarrow n_1 \quad \vdash e_2 \Rightarrow n_2}{\vdash e_1 + e_2 \Rightarrow n_1 + n_2} \quad \text{MUL} \frac{\vdash e_1 \Rightarrow n_1 \quad \vdash e_2 \Rightarrow n_2}{\vdash e_1 * e_2 \Rightarrow n_1 \times n_2}$$

```
type Value = BigInt
def interp(expr: Expr): Value = expr match
  case Num(n)      => n
  case Add(l, r)   => interp(l) + interp(r)
  case Mul(l, r)   => interp(l) * interp(r)
```

We will implement the **interpreter** for AE according to the following **big-step operational (natural) semantics**:

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$$\text{NUM} \frac{}{\vdash n \Rightarrow n} \quad \text{ADD} \frac{\vdash e_1 \Rightarrow n_1 \quad \vdash e_2 \Rightarrow n_2}{\vdash e_1 + e_2 \Rightarrow n_1 + n_2} \quad \text{MUL} \frac{\vdash e_1 \Rightarrow n_1 \quad \vdash e_2 \Rightarrow n_2}{\vdash e_1 * e_2 \Rightarrow n_1 \times n_2}$$

```
type Value = BigInt
def interp(expr: Expr): Value = expr match
  case Num(n)      => n
  case Add(l, r) => interp(l) + interp(r)
  case Mul(l, r) => interp(l) * interp(r)
```

```
interp(Expr("42"))      // interp(Num(42))                = 42
interp(Expr("-1+7"))    // interp(Add(Num(-1), Num(7)))    = 6
interp(Expr("1+2*3"))   // interp(Add(Num(1), Mul(Num(2), Num(3)))) = 7
```

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2. Interpreters

<https://github.com/ku-plrg-classroom/docs/tree/main/cose212/ae>

- Please see above document on GitHub:
 - Implement `interp` function.
 - Implement `countNums` function.
- It is just an exercise, and you **don't need to submit** anything.
- However, some exam questions might be related to this exercise, so we **strongly recommend** solving it by yourself.

- Identifiers (1)

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