Midterm Exam

COSE215: Theory of Computation
2023 Spring

Instructor: Jihyeok Park

April 24, 2023. 14:00-15:15

- **If you are not good at English, please write your answers in Korean.**
  (영어가 익숙하지 않은 경우, 답안을 한글로 작성해 주세요.)
- **Write answers in good handwriting.**
  If we cannot recognize your answers, you will not get any points.
  (글씨를 알아보기 힘들면 점수를 드릴 수 없습니다. 답안을 읽기 좋게 작성해주세요.)
- **Write your answers in the boxes provided.**
  (답안을 제공된 박스 안에 작성해 주세요.)

Student ID	
Student Name	

Question:	1	2	3	4	5	6	7	Total
Points:	10	20	10	20	15	15	10	100
Score:								

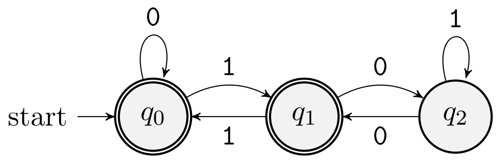
1. A **deterministic finite automaton (DFA)** D is 5-tuple:

$$D = (Q, \Sigma, \delta, q_0, F)$$

- Q is a finite set of **states**
- Σ is a finite set of **symbols**
- $\delta : Q \times \Sigma \to Q$ is the **transition function**
- $q_0 \in Q$ is the **initial state**
- $F \subseteq Q$ is the set of **final states**

Consider the following DFA D described by a **transition diagram**:

$D =$



(a) 4 points Fill in the blanks in the **transition table** of D.
(Note that $\to$ indicates the **initial state**, and $*$ indicates a **final state**.)

	$q_i \in Q$	0	1
	q_0	q_0	
	q_1		
	q_2		

(b) 4 points The **extended transition function** $\delta^* : Q \times \Sigma^* \to Q$ of D is defined as:

- **(Basis Case)** $\delta^*(q, \epsilon) = q$
- **(Induction Case)** $\delta^*(q, aw) = \delta^*(\delta(q, a), w)$

Fill in the blanks in the results of the **extended transition function** δ^* of D.

$\delta^*(q_0, \epsilon) =$ ☐ $\quad\delta^*(q_0, \texttt{1}) =$ ☐ $\quad\delta^*(q_1, \texttt{010}) =$ ☐ $\quad\delta^*(q_2, \texttt{00101}) =$ ☐

(c) 2 points Describe the **language** accepted by D:

$$L(D) = \left\{ \qquad \qquad \right\}$$

(Hint: you can use the notation $\mathbb{N}(w)$ which denotes the natural number equivalent to w in binary, allowing leading zeros. For example, $\mathbb{N}(\texttt{1}) = 1$, $\mathbb{N}(\texttt{101}) = 5$, $\mathbb{N}(\texttt{000101}) = 5$, and $\mathbb{N}(\texttt{000}) = 0$.)

2. Design a **DFA** using a **transition diagram** that accepts each of the following languages.

(a) 5 points $L = \{\texttt{0}^k \mid k \geq 2\}$

(b) 5 points $L = \{\texttt{a}w\texttt{b} \mid w \in \{\texttt{a}, \texttt{b}\}^*\}$

(c) 5 points $L = \{w \in \{\texttt{a}, \texttt{b}\}^* \mid N_{\texttt{a}}(w) \equiv 1 \pmod 2 \ \wedge \ N_{\texttt{b}}(w) \equiv 1 \pmod 2 \}$

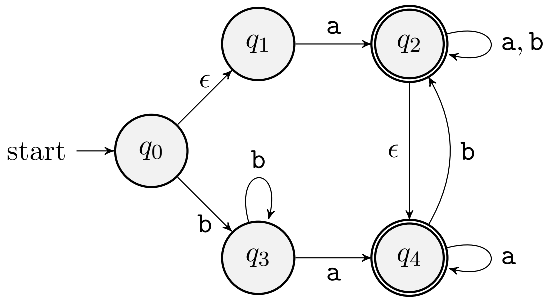
(Note that $N_{\texttt{a}}(w)$ and $N_{\texttt{b}}(w)$ denotes the number of a and b in w, respectively. For example, $N_{\texttt{a}}(\texttt{abbab}) = 2$ and $N_{\texttt{b}}(\texttt{abbab}) = 3$.)

(d) 5 points $L = \{w \in \{\texttt{0}, \texttt{1}\}^* \mid \mathbb{N}(w) \equiv 2 \pmod 5\}$.

(Note that $\mathbb{N}(w)$ represents the natural number equivalent to w in binary, allowing leading zeros. For example, $\mathbb{N}(\texttt{1}) = 1$, $\mathbb{N}(\texttt{101}) = 5$, $\mathbb{N}(\texttt{000101}) = 5$, and $\mathbb{N}(\texttt{000}) = 0$.)

3. Consider the following **ϵ-nondeterministic finite automaton (ϵ-NFA)** N_ϵ:

$$N_\epsilon = (Q = \{q_0, q_1, q_2, q_3, q_4\}, \Sigma = \{\texttt{a}, \texttt{b}\}, \delta, q_0, F = \{q_2, q_4\})$$



(a) [4 points] Construct a **DFA** D using a **transition table** such that $L(D) = L(N_\epsilon)$ via the **subset construction**.
(Note that the states P in D should be the **subsets** of states Q in N_ϵ. And, $\rightarrow$ indicates the **initial state**, and $*$ indicates a **final state**. Consider only **reachable states** of D when constructing the transition table.)

	$p_i \in P = \mathcal{P}(Q)$	a	b
	$p_0 = \{ \; q_0, \; q_1 \; \}$		p_2
	$p_1 = \{ \; \}$		
	$p_2 = \{ \; q_3 \; \}$		
	$p_3 = \{ \; \}$		

(b) [4 points] Fill in the table in the left side using the **table-filling algorithm**, and define the **equivalence classes** $P/_\equiv$ in the right side using the result of the algorithm.

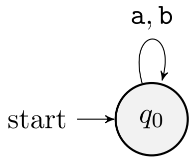
p_1			
p_2			
p_3			
	p_0	p_1	p_2

$P/_\equiv = \{$
$\quad p'_0 = \{ \quad \},$
$\quad p'_1 = \{ \quad \},$
$\}$

(c) [2 points] Construct a **minimized DFA** D^M of D.
(Note that the states of D^M should be $P/_\equiv = \{p'_0, p'_1\}$)

4. Write a **regular expressions (RE)** that represents each of the following languages:

(a) 4 points The language of the following DFA.

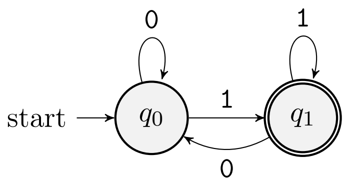


$R =$

(b) 4 points $L = \{(\mathtt{ab})^n \mathtt{c}^m \mid n, m \geq 0\}$.

$R =$

(c) 4 points The language of the following DFA.



$R =$

(d) 4 points $L = \{w \in \{\mathtt{0}, \mathtt{1}\}^* \mid w \text{ has an even number of } \mathtt{0}\text{'s}\}$.

$R =$

(e) 4 points $L = \{\mathtt{a}^n \mathtt{b}^m \mid n + 2m \equiv 0 \pmod 3\ \}$.

$R =$

5. Fill in the blanks in the **proof** showing that each of the following languages is **not regular**.

(a) 5 points $L = \{\mathtt{a}^n\mathtt{b}\mathtt{c}^n \mid n \geq 0\}$.

We can prove L is not regular by using the **pumping lemma** for regular languages.

1. Assume that any positive integer n is given. (i.e., $n \geq 1$)
2. Pick a word $L \ni w =$ ________ .
3. $|w| =$ ________ $\geq n$.
4. Assume that any split $w = xyz$ satisfying ① $|y| > 0$ and ② $|xy| \leq n$ is given.
5. Let $i =$ ________ . We need to show that ¬③ $xy^iz \notin L$:

(b) 10 points $L = \{\mathtt{a}^p \mid p \text{ is a prime number } \}$.

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6. Design a **context-free grammar** that represents each of the following languages.
   (a) 5 points $L = L(R)$ where $R = \texttt{(0|1(01}^*\texttt{0)}^*\texttt{1)}^*$.

   (b) 5 points $L = \{\texttt{a}^n\texttt{b}^m\texttt{c}^m\texttt{d}^n \mid n, m \geq 0\}$

   (c) 5 points $L = \{w \in \{\texttt{a}, \texttt{b}, \texttt{c}\}^* \mid N_{\texttt{a}}(w) = N_{\texttt{b}}(w) - N_{\texttt{c}}(w)\}$

7. 10 points **True/False questions.** Please answer **O** for True and **X** for False. (Note that each question is worth **1 points**, but you will get **-1 points** for each wrong answer. So, if you are unsure about the answer, please leave it blank.)

   1. The Kleene star of the empty language is the empty language.
   2. Every regular language is context-free.
   3. We can prove that a language is regular using the pumping lemma.
   4. $L = \{w \in \{\varnothing, \varepsilon, \texttt{a}, \texttt{b}, \texttt{|}, \texttt{*}, \texttt{(}, \texttt{)}\}^* \mid w \text{ is a regular expression } \}$ is regular.
   5. Regular languages are closed under the union but not under the intersection.
   6. Regular languages are closed under the complement and the homomorphism.
   7. We can construct two parse trees having the same yield for an ambiguous CFG.
   8. All words in an ambiguous CFG have at least two left-most derivations.
   9. We can always remove the ambiguity in a CFG.
   10. Every CFG for an inherently ambiguous language is ambiguous.

This is the last page.
I hope that your tests went well!