Lecture 15 – Examples of Pushdown Automata

COSE215: Theory of Computation

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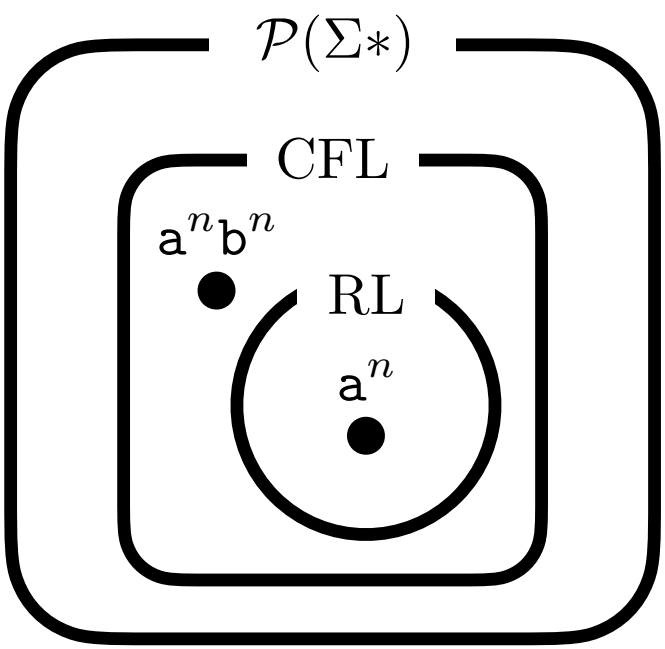
Recall

A **pushdown automaton (PDA)** is a finite automaton with a **stack**.

- Acceptance by **final states**
- Acceptance by **empty stacks**



Languages	**Automata**	**Grammars**
Context-Free Language **(CFL)**	Pushdown Automata **(PDA)**	Context-Free Grammar **(CFG)**
Regular Language **(RL)**	Finite Automata **(FA)**	Regular Expression **(RE)**

Contents

Example 1: $a^n b^n$

PLRG

Construct a PDA that accepts the language by final states:

$$L_F(P) = \{a^n b^n \mid n \geq 0\}$$

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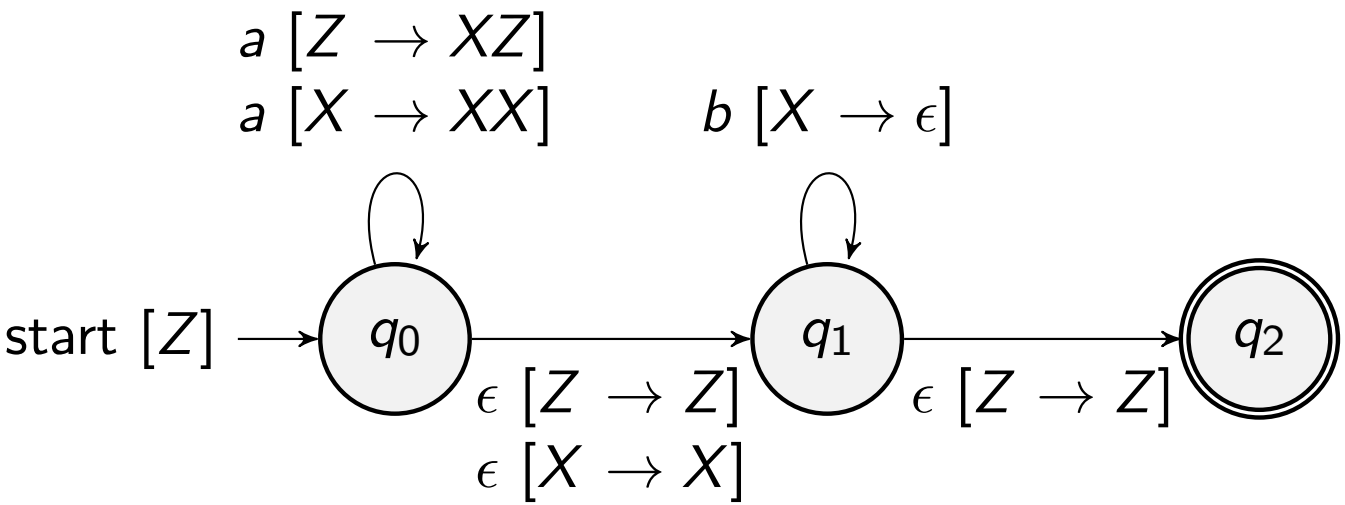

Example 2: $a^n b^{2n}$

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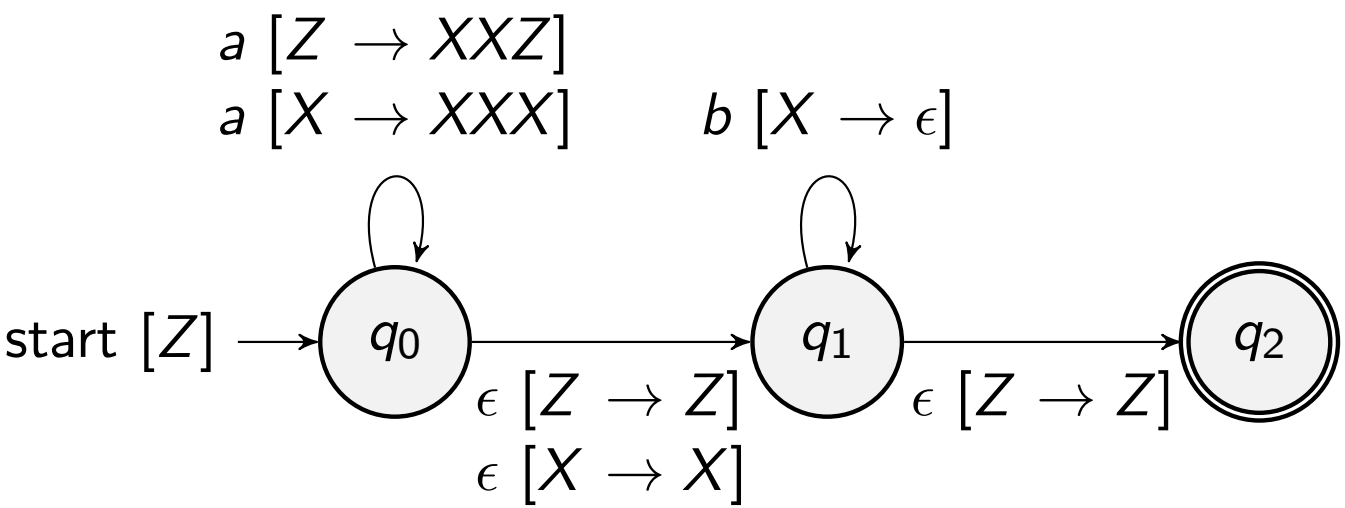

Example 3: ww^R

PLRG

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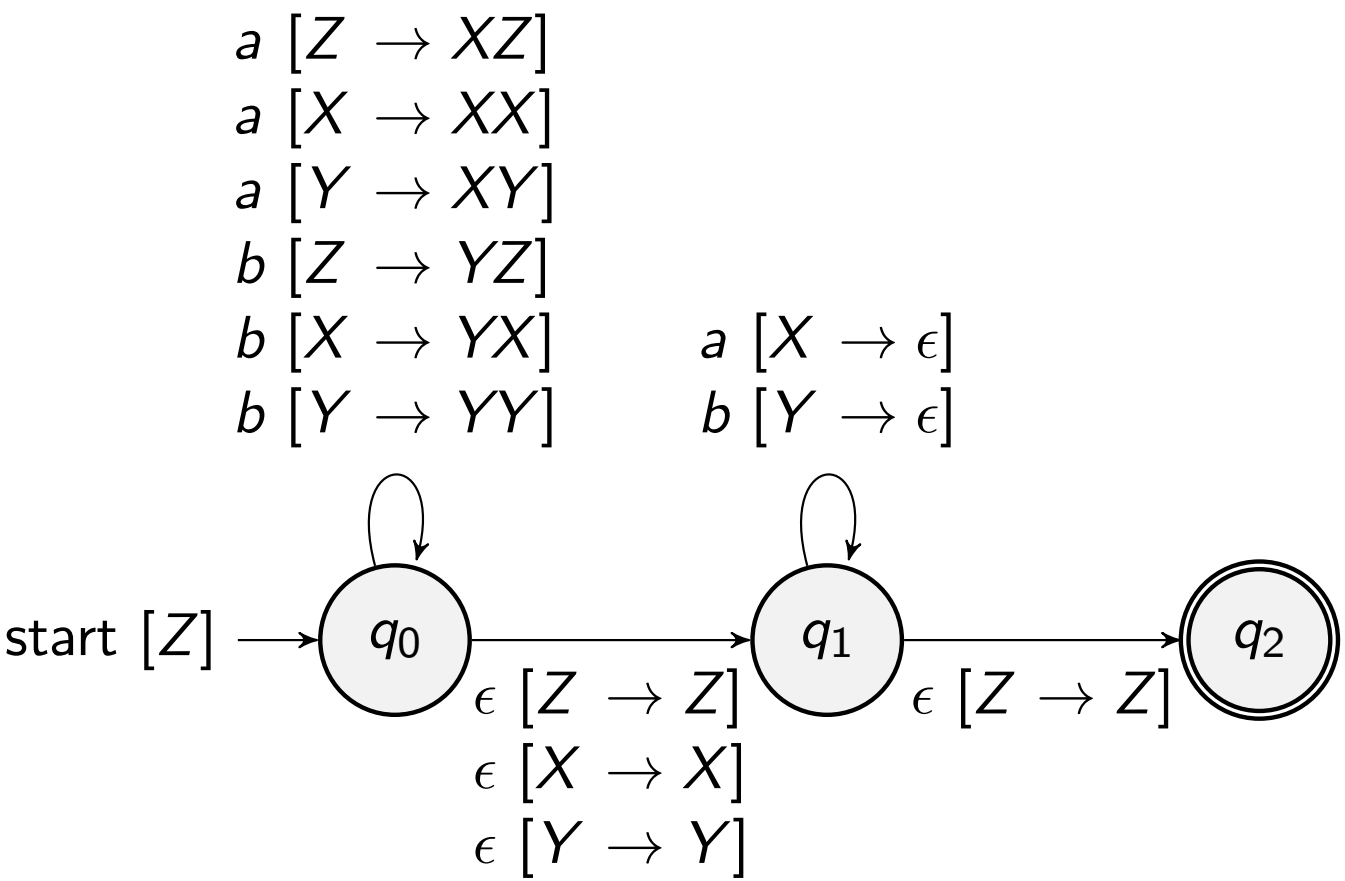

Example 4: Equal Number of a's and b's

PLRG

Construct a PDA that accepts the language by empty stacks:

$$L_E(P) = \{w \in \{\mathtt{a}, \mathtt{b}\}^* \mid N_{\mathtt{a}}(w) = N_{\mathtt{b}}(w)\}$$

where $N_{\mathtt{a}}(w)$ and $N_{\mathtt{b}}(w)$ are the number of a's and b's in w, respectively.

Example 4: Equal Number of a's and b's

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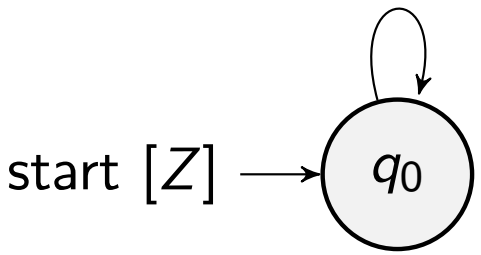
$$\begin{aligned}
&a\ [Z \to PZ]\\
&a\ [P \to PP]\\
&a\ [N \to \epsilon]\\
&b\ [Z \to NZ]\\
&b\ [P \to \epsilon]\\
&b\ [N \to NN]\\
&\epsilon\ [Z \to \epsilon]
\end{aligned}$$



Example 5: Unequal Number of a's and b's

PLRG

Construct a PDA that accepts the language by empty stacks:

$$L_E(P) = \{w \in \{\mathtt{a}, \mathtt{b}\}^* \mid N_{\mathtt{a}}(w) \neq N_{\mathtt{b}}(w)\}$$

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Example 5: Unequal Number of a's and b's

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$a\ [P \to PP]$
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$\epsilon\ [P \to \epsilon]$
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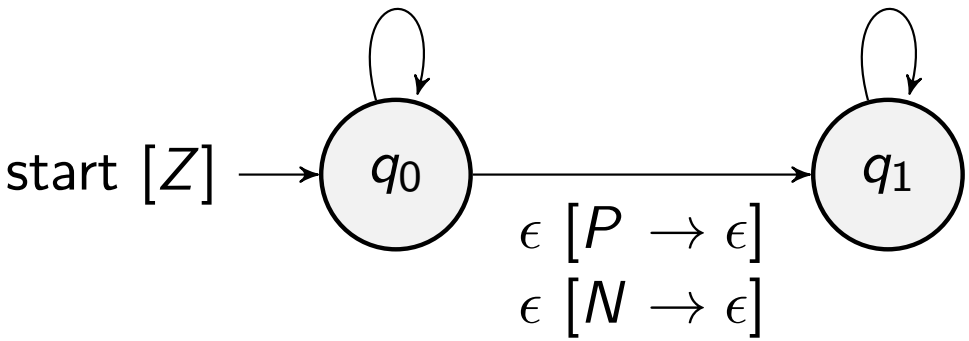

Example 6: Not of the Form *ww*

PLRG

Construct a PDA that accepts the language by empty stacks:

$$L_E(P) = \{x \in \{\mathtt{a}, \mathtt{b}\}^* \mid x \text{ is not of the form } ww\}$$

Example 6: Not of the Form *ww*

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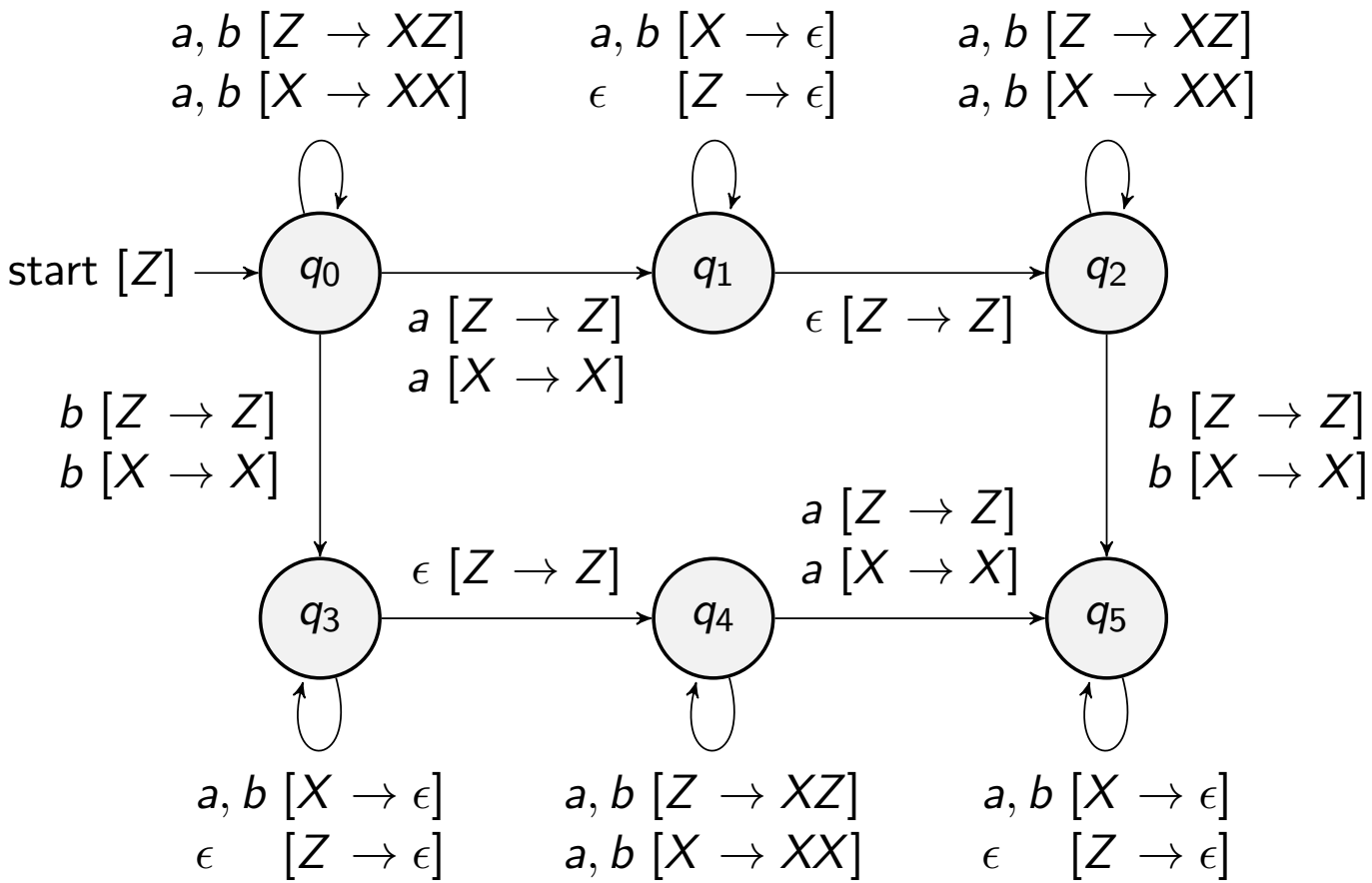

Summary

PLRG

1. Examples of Pushdown Automata
    - Example 1: $\mathtt{a}^n\mathtt{b}^n$
    - Example 2: $\mathtt{a}^n\mathtt{b}^{2n}$
    - Example 3: ww^R
    - Example 4: Equal Number of $\mathtt{a}$'s and $\mathtt{b}$'s
    - Example 5: Unequal Number of $\mathtt{a}$'s and $\mathtt{b}$'s
    - Example 6: Not of the Form ww

Next Lecture

- Equivalence of Pushdown Automata and Context-Free Grammars

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