

# Lecture 18 – Normal Forms of Context-Free Grammars

## COSE215: Theory of Computation

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2023 Spring

- A **context-free grammar (CFG)** is a 4-tuple:

$$G = (V, \Sigma, S, R)$$

where

- $V$ : a finite set of **variables** (nonterminals)
  - $\Sigma$ : a finite set of **symbols** (terminals)
  - $S \in V$ : the **start variable**
  - $R \subseteq V \times (V \cup \Sigma)^*$ : a set of **production rules**.
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- How to **simplify** a CFG?

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- How to **simplify** a CFG?

Let's put it in **Chomsky normal form (CNF)**!

## Definition (Chomsky Normal Form)

A CFG is in **Chomsky normal form (CNF)** if all productions are of the form for some  $A, B, C \in V$  and  $a \in \Sigma$ :

$$A \rightarrow BC \quad \text{OR} \quad A \rightarrow a$$

(If  $\epsilon \in L(G)$ , then  $S \rightarrow \epsilon$  is allowed with forbidden  $S$  on RHSs.)

$$S \rightarrow 0ABC \mid 1B \mid BB$$

$$A \rightarrow ABB0 \mid C$$

$$B \rightarrow 0B \mid 1$$

$$C \rightarrow CC \mid \epsilon$$

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Is it possible to put this CFG in CNF?

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Is it possible to put this CFG in CNF? **Yes!**

$$S \rightarrow XS_1 \mid XB \mid YB \mid BB \quad A \rightarrow AA_1 \mid BA_2 \quad B \rightarrow XB \mid 1$$

$$S_1 \rightarrow AB$$

$$A_1 \rightarrow BA_2$$

$$X \rightarrow 0$$

$$A_2 \rightarrow BX$$

$$Y \rightarrow 1$$

## 1. Chomsky Normal Form (CNF)

- Eliminating  $\epsilon$ -Productions

  - Nullable Variables

- Eliminating Unit Productions

  - Unit Pairs

- Eliminating Useless Variables

  - Generating Variables

  - Reachable Variables

- Putting CFG in CNF

# Eliminating $\epsilon$ -Productions

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Let's construct a new CFG  $G'$  from  $G$  such that

$$L(G') = L(G) \setminus \{\epsilon\}$$

by eliminating  $\epsilon$ -productions:

- 1 Find all **nullable variables**.
- 2 Construct a new CFG with productions produced by replacing nullable variables with  $\epsilon$  in all combinations, except for the  $\epsilon$ -production.

## Definition (Nullable Variables)

For a given CFG  $G = (V, \Sigma, S, R)$ , a variable  $A \in V$  is **nullable** if

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We can inductively define the set of **nullable variables**:

- **(Basis Case)** If  $A \rightarrow \epsilon \in R$ , then  $A$  is nullable.
- **(Induction Case)** If  $A \rightarrow X_1 X_2 \cdots X_n \in R$  and  $X_1, X_2, \dots, X_n$  are all nullable, then  $A$  is nullable.

Consider the following CFG:

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# Eliminating Unit Productions

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Yes, we can do it by following the steps below:

- 1 Find all **unit pairs**.
- 2 Construct a new CFG by adding all (recursively) possible non-unit productions of  $B$  to  $A$  for each unit pair  $(A, B)$ .

## Definition (Unit Pairs)

For a given CFG  $G = (V, \Sigma, S, R)$ , a pair of variables  $(A, B) \in V \times V$  is a **unit pair** if

$$A \Rightarrow^* B$$

### Definition (Unit Pairs)

For a given CFG  $G = (V, \Sigma, S, R)$ , a pair of variables  $(A, B) \in V \times V$  is a **unit pair** if

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We can inductively define the set of **unit pairs**:

- **(Basis Case)**  $(A, A)$  is a unit pair for all  $A \in V$ .
- **(Induction Case)** If  $(A, B)$  is a unit pair and  $B \rightarrow C \in R$ , then  $(A, C)$  is a unit pair.

## Eliminating Unit Productions – Example

After eliminating  $\epsilon$ -productions:

$$S \rightarrow 0ABC \mid 0BC \mid 0AB \mid 0B \mid 1B \mid BB$$

$$A \rightarrow ABB0 \mid BB0 \mid C$$

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- 1 Find all **unit pairs**:

$$\{(S, S), (A, A), (A, C), (B, B), (C, C), (D, D), (D, A), (D, C)\}$$

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What are useless variables?

- **Non-generating variables:** Variables that cannot derive any word.
- **Unreachable variables:** Variables unreachable from the start variable.

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Yes, we can do it by following the steps below:

- 1 Find all **generating variables**.
- 2 Find all **reachable variables**.
- 3 Construct a new CFG by removing all productions that contain non-generating variables or come from unreachable variables.



## Definition (Generating Variables)

For a given CFG  $G = (V, \Sigma, S, R)$ , a variable  $A \in V$  is a **generating variable** if for some  $w \in \Sigma^*$ ,

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We can inductively define the set of **generating variables**:

- **(Basis Case)** There is no basis case.
- **(Induction Case)** If  $A \rightarrow \alpha \in R$  and  $\alpha$  contains only symbols or generating variables, then  $A$  is a generating variable.

## Definition (Reachable Variables)

For a given CFG  $G = (V, \Sigma, S, R)$ , a variable  $A \in V$  is a **reachable variable** if there exists a derivation:

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## Definition (Reachable Variables)

For a given CFG  $G = (V, \Sigma, S, R)$ , a variable  $A \in V$  is a **reachable variable** if there exists a derivation:

$$S \Rightarrow^* \alpha A \beta$$

We can inductively define the set of **reachable variables**:

- **(Basis Case)** The start variable  $S$  is reachable variable.
- **(Induction Case)** If  $A \in V$  is a reachable variable and  $A \rightarrow \alpha \in R$ , then all variables in  $\alpha$  are reachable variables.

## Eliminating Useless Variables – Example

After eliminating  $\epsilon$ -productions and unit productions:

$$S \rightarrow 0ABC \mid 0BC \mid 0AB \mid 0B \mid 1B \mid BB$$

$$A \rightarrow ABB0 \mid BB0 \mid CC$$

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- 1 Find all **generating variables**:  $\{S, A, B, D\}$  –  $C$  is non-generating.
- 2 Find all **reachable variables**:

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- 2 Find all **reachable variables**:  $\{S, A, B, C\}$  –  $D$  is unreachable.
- 3 Construct a new CFG by removing all productions that contain non-generating variables or come from unreachable variables:

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Our goal is to put a CFG in **Chomsky normal form (CNF)** consisting of:

$$A \rightarrow BC \quad \text{OR} \quad A \rightarrow a$$

(If  $\epsilon \in L(G)$ , then  $S \rightarrow \epsilon$  is allowed with forbidden  $S$  on RHSs.)

We can put a CFG in CNF by following the steps below:

- ① If  $S$  on RHSs, add a new start variable  $S'$  and a production  $S' \rightarrow S$ .
- ② Eliminate  $\epsilon$ -productions, unit productions, and useless variables.
- ③ Arrange so that all RHSs whose length is greater than 1 consist only of variables. To do so, if terminal  $a$  appears in a RHS, then replace it with a new variable  $A$  and add a production  $A \rightarrow a$ .
- ④ Replace all RHSs whose length is greater than 2 with a chain of variables. To do so, if  $A \rightarrow X_1X_2 \cdots X_n$  is a production with  $n > 2$ , then replace it with a sequence of productions:

$$A \rightarrow X_1A_1 \quad A_1 \rightarrow X_2A_2 \quad \cdots \quad A_{n-2} \rightarrow X_{n-1}X_n$$

- ⑤ If  $\epsilon$  is in the original CFG, add a production  $S \rightarrow \epsilon$  (or  $S' \rightarrow \epsilon$ ).

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- 3 Arrange so that all RHSs whose length  $> 1$  consist only of variables:

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- 3 Arrange so that all RHSs whose length  $> 1$  consist only of variables:

$$S \rightarrow XAB \mid XB \mid YB \mid BB \quad X \rightarrow 0$$

$$A \rightarrow ABBX \mid BBX \quad Y \rightarrow 1$$

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- 4 Replace all RHSs whose length  $> 2$  with a chain of variables:

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- 2 Eliminate  $\epsilon$ -productions, unit productions, and useless variables:

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- 4 Replace all RHSs whose length  $> 2$  with a chain of variables:

$$\begin{aligned} S &\rightarrow XS_1 \mid XB \mid YB \mid BB & A &\rightarrow AA_1 \mid BA_2 & B &\rightarrow XB \mid 1 \\ S_1 &\rightarrow AB & A_1 &\rightarrow BA_2 & X &\rightarrow 0 \\ & & A_2 &\rightarrow BX & Y &\rightarrow 1 \end{aligned}$$

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- 4 Replace all RHSs whose length  $> 2$  with a chain of variables:

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- 5 If  $\epsilon$  is in the original CFG, add a production  $S \rightarrow \epsilon$  (or  $S' \rightarrow \epsilon$ ): **No.**

## Putting CFG in CNF – Example 2

Let's put the following CFG in CNF:

$$S \rightarrow aSb \mid \epsilon$$

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- 2 Eliminate  $\epsilon$ -productions, unit productions, and useless variables:

$$S' \rightarrow aSb \mid ab \quad S \rightarrow aSb \mid ab$$

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- ② Eliminate  $\epsilon$ -productions, unit productions, and useless variables:

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- ③ Arrange so that all RHSs whose length  $> 1$  consist only of variables:

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$$S' \rightarrow AS_1 \mid AB \quad S \rightarrow AS_1 \mid AB \quad S_1 \rightarrow SB \quad A \rightarrow a \quad B \rightarrow b$$

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## 1. Chomsky Normal Form (CNF)

- Eliminating  $\epsilon$ -Productions

  - Nullable Variables

- Eliminating Unit Productions

  - Unit Pairs

- Eliminating Useless Variables

  - Generating Variables

  - Reachable Variables

- Putting CFG in CNF

- Properties of Context-Free Languages

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