Lecture 22 – Examples of Turing Machines

COSE215: Theory of Computation

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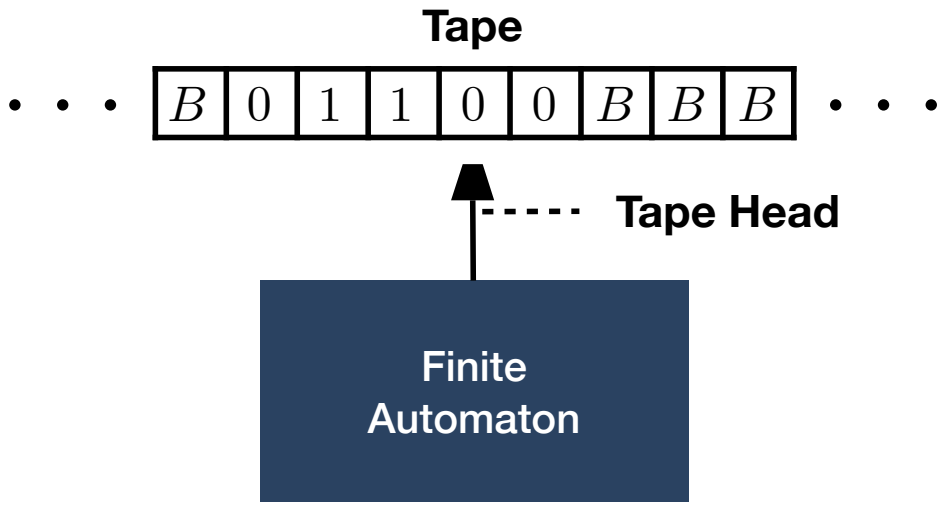
Recall



- A **Turing machine (TM)** is a finite automaton with a **tape**:
  1. A **finite automaton** with a deterministic transition function.
  2. A **tape** is a one-dimensional infinite array of cells.
     - Each cell contains a **tape symbol**.
     - The **blank symbol** B is a special symbol representing an empty cell.
  3. A **tape head** is a device that can read and write symbols on the tape.
     - It can move **left** or **right** one cell at a time.
- We can use Turing machines as **computing machines**.

Contents

Example 1: $L = \{a^n b^n c^n \mid n \geq 0\}$

PLRG

Construct a Turing machine that accepts the language:

$$L(M) = \{a^n b^n c^n \mid n \geq 0\}$$

···	B	a	a	b	b	c	c	B	···

1: **while** there are a's **do**
2: Find and Replace a with X
3: Find and Replace b with Y
4: Find and Replace c with Z
5: **end while**
6: Check if only Y's and Z's are left

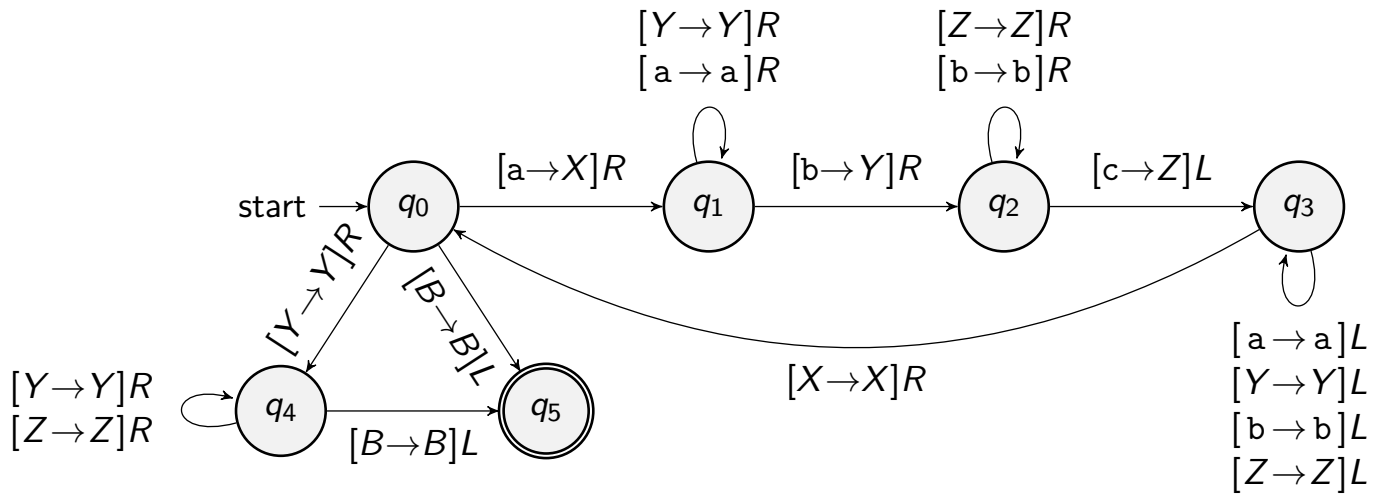
Example 1: $L = \{a^n b^n c^n \mid n \geq 0\}$

PLRG

Construct a Turing machine that accepts the language:

$$L(M) = \{a^n b^n c^n \mid n \geq 0\}$$

See the example for aabbcc $\in L(M)$.[1]



[1] https://plrg.korea.ac.kr/courses/cose215/materials/tm-an-bn-cn.pdf

Example 2: $L = \{ww \mid w \in \{\mathtt{a}, \mathtt{b}\}^*\}$

PLRG

Construct a Turing machine that accepts the language:

$$L(M) = \{ww \mid w \in \{\mathtt{a}, \mathtt{b}\}^*\}$$

⋯	B	a	b	b	a	b	b	B	⋯

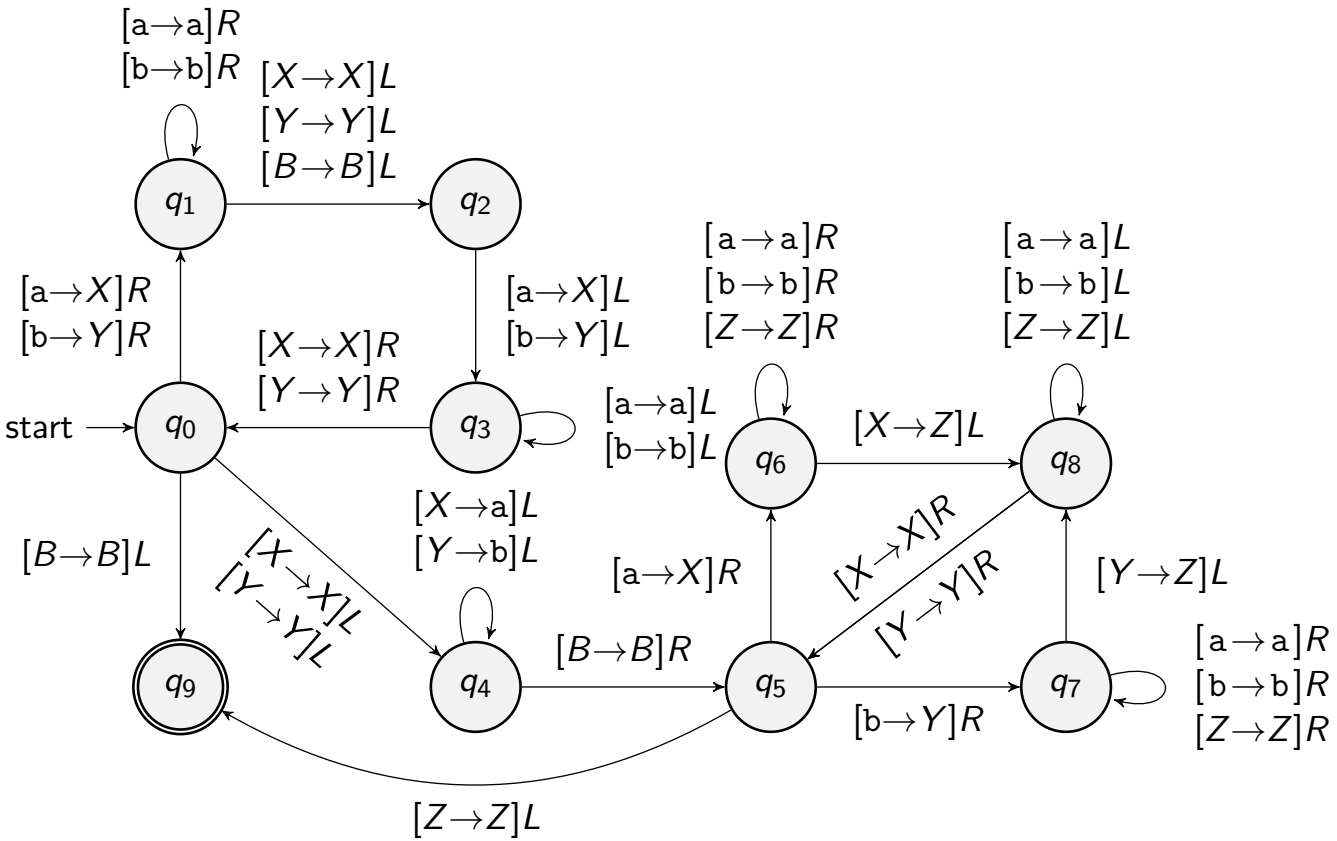
1: Find the middle of the input
2: Replace all X's (or Y's) with a's (or b's) in the first half
3: **while** there are input symbols in the first half **do**
4: Replace a (or b) with X (or Y) in the first half
5: Find and Replace matched X (or Y) with Z in the second half
6: **end while**

Example 2: $L = \{ww \mid w \in \{a, b\}^*\}$

Construct a Turing machine that accepts the language:

$$L(M) = \{ww \mid w \in \{a, b\}^*\}$$

See the example for abbabb $\in L(M)$.[2]



[2] https://plrg.korea.ac.kr/courses/cose215/materials/tm-w-w.pdf

Example 3: $L = \{a^i b^j c^{i \times j} \mid i, j \geq 0\}$

PLRG

Construct a Turing machine that accepts the language:

$$L(M) = \{a^i b^j c^{i \times j} \mid i, j \geq 0\}$$

···	B	a	a	b	b	b	c	c	c	c	c	c	B	···

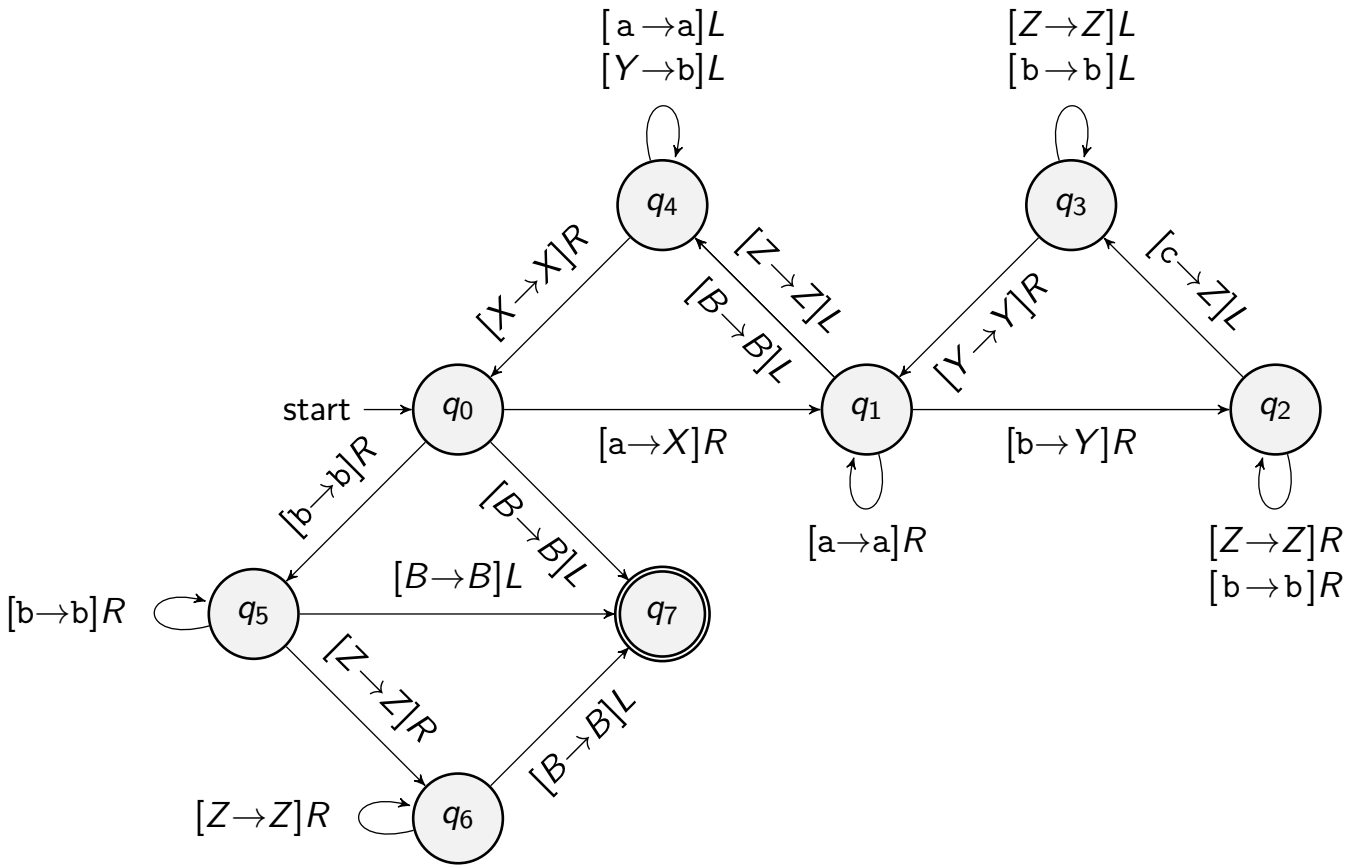
1: **while** there are a's **do**
2: Find and Replace a with X
3: **while** there are b's **do**
4: Find and Replace b with Y
5: Find and Replace c with Z
6: **end while**
7: Roll back all Y's to b's
8: **end while**
9: Check if only b's and Z's are left

Example 3: $L = \{a^i b^j c^{i \times j} \mid i, j \geq 0\}$

Construct a Turing machine that accepts the language:

$$L(M) = \{a^i b^j c^{i \times j} \mid i, j \geq 0\}$$

See the example for $aabbbccccccc \in L(M)$.[3]



[3] https://plrg.korea.ac.kr/courses/cose215/materials/tm-ai-bj-cij.pdf

Example 4: Flip Bits – $f(w) =$ (flip of w)

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Construct a Turing machine that computes the function:

$$f(w) = \text{(the flip of each bit in } w\text{)}$$

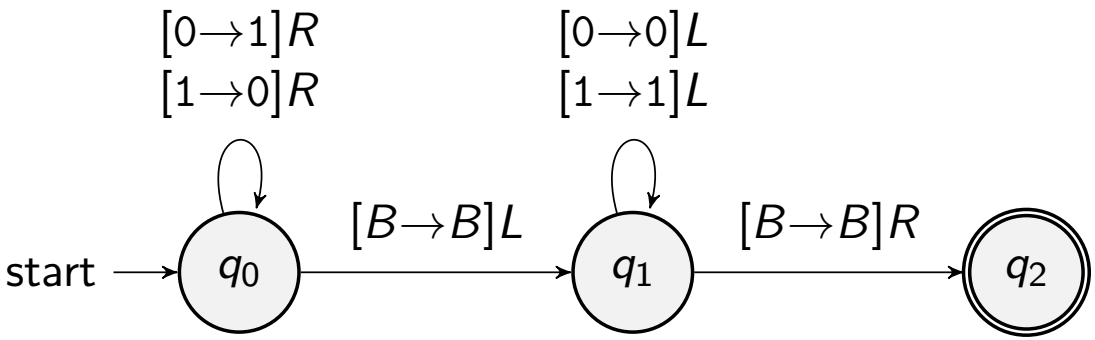
···	B	1	0	1	1	1	0	0	B	···

1: Flip each bit of the input: $1 \to 0$ and $0 \to 1$
2: Go to the first input symbol

Example 4: Flip Bits – $f(w) =$ (flip of w)

$$f(w) = \text{(the flip of each bit in } w\text{)}$$

See the example for $f(1011100) = 0100011$.[4]



[4] https://plrg.korea.ac.kr/courses/cose215/materials/tm-flip.pdf

Example 5: Unary Addition – $f(a, b) = a + b$

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Construct a Turing machine that computes the function:

$$f(a, b) = a + b \qquad \text{where} \qquad a \text{ and } b \text{ are unary numbers}$$

···	B	1	1	1	+	1	1	B	···

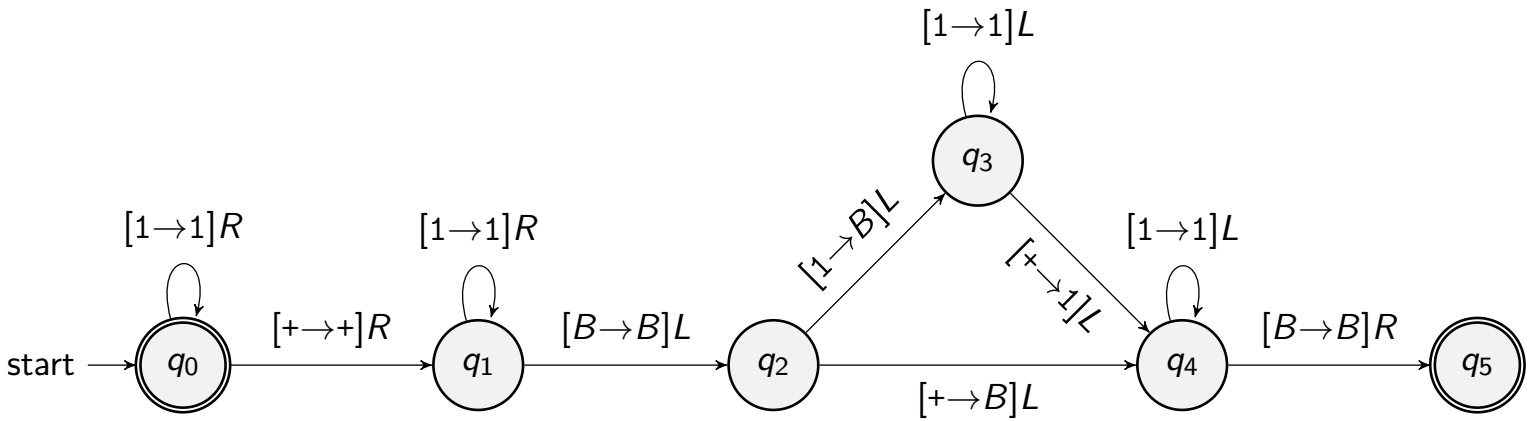
1: Find + after 1's
2: **if** the last symbol is 1 **then**
3: Find and Remove the last 1
4: Find and Replace the + with 1
5: **else**
6: Remove the +
7: **end if**
8: Go to the first input symbol

Example 5: Unary Addition – $f(a, b) = a + b$

Construct a Turing machine that computes the function:

$$f(a, b) = a + b \qquad \text{where} \qquad a \text{ and } b \text{ are unary numbers}$$

See the example for $f(111{+}11) = 11111$.[5]



[5] https://plrg.korea.ac.kr/courses/cose215/materials/tm-unary-add.pdf

Example 6: Data Copy – $f(w) = ww$

Construct a Turing machine that computes the function:

$$f(w) = ww \qquad \text{where} \qquad w \in \{0, 1\}^*$$

⋯	B	0	1	1	B	B	B	B	⋯

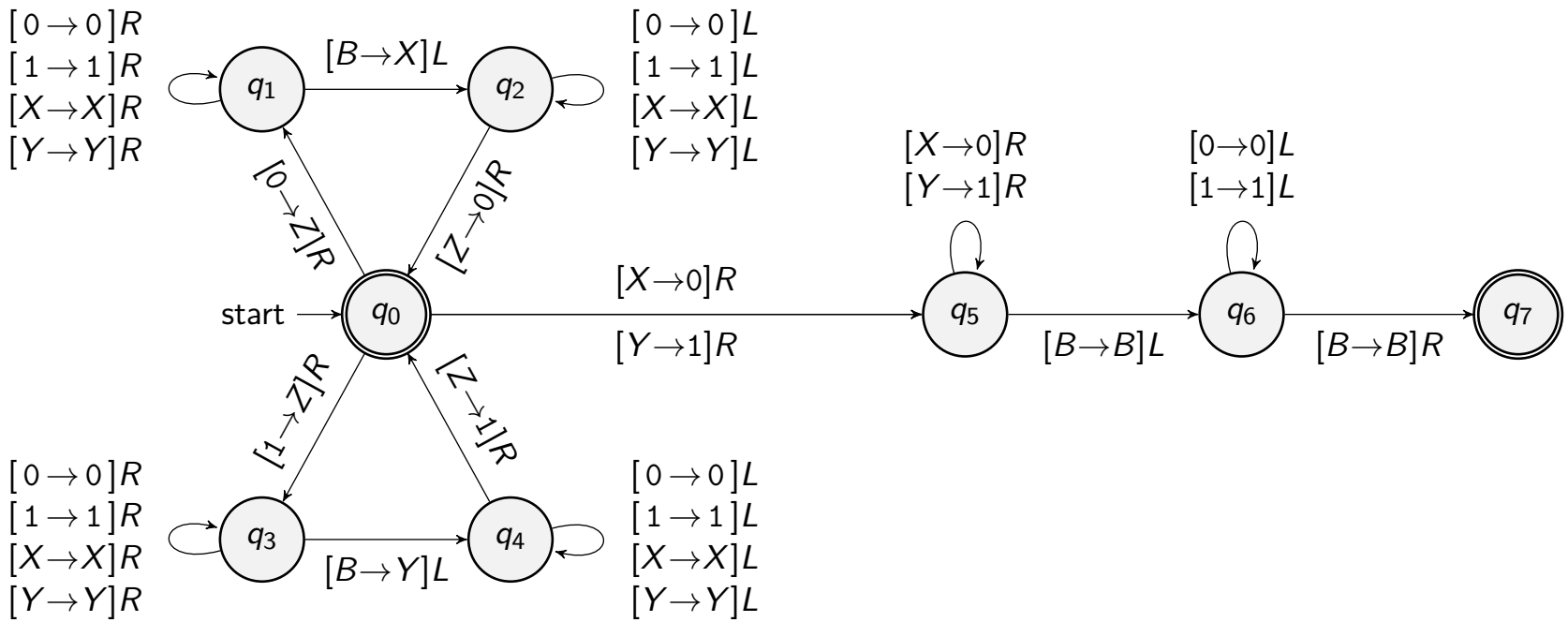
1: **while** there are input symbols **do**
2: Find and Replace 0 (or 1) with Z
3: Find and Fill the first blank with X (or Y) for 0 (or 1)
4: Roll back Z to the original 0 (or 1)
5: **end while**
6: Replace X's and Y's with 0's and 1's
7: Go to the first input symbol

Example 6: Data Copy – $f(w) = ww$

Construct a Turing machine that computes the function:

$$f(w) = ww \qquad \text{where} \qquad w \in \{0,1\}^*$$

See the example for $f(011) = 011011$.[6]



[6] https://plrg.korea.ac.kr/courses/cose215/materials/tm-data-copy.pdf

Summary

PLRG

1. Turing Machines as Word Recognizers
   - Example 1: $L = \{\mathtt{a}^n\mathtt{b}^n\mathtt{c}^n \mid n \geq 0\}$
   - Example 2: $L = \{ww \mid w \in \{\mathtt{a}, \mathtt{b}\}^*\}$
   - Example 3: $L = \{\mathtt{a}^i\mathtt{b}^j\mathtt{c}^{i\times j} \mid i, j \geq 0\}$

2. Turing Machines as Computing Machines
   - Example 4: Flip Bits – $f(w) =$ (flip of w)
   - Example 5: Unary Addition – $f(a, b) = a + b$
   - Example 6: Data Copy – $f(w) = ww$

Next Lecture

- Extensions of Turing Machines

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