Lecture 23 – Extensions of Turing Machines

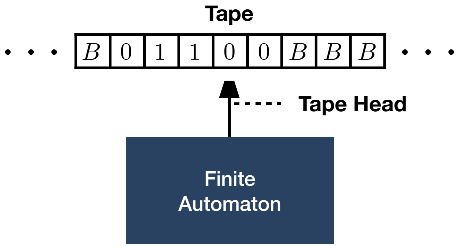
COSE215: Theory of Computation

Jihyeok Park

PLRG

2023 Spring

Recall

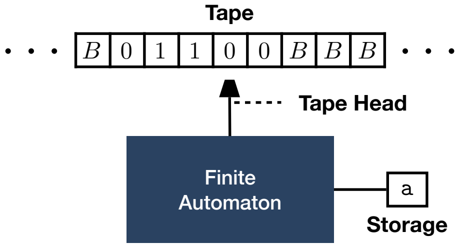


- A **Turing machine (TM)** is a finite automaton with a **tape**.
- A language accepted by a TM is **Recursively Enumerable**.
- What happens if we define **other extensions** of TMs?
- Are they **more powerful** than TMs? **NO!!**

Contents

TMs with Storage

We can define a TM with a **storage**:



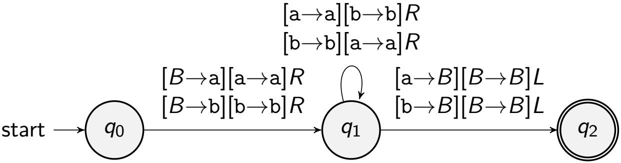
It has additional **storage** affecting the transition function:

$$\delta : Q \times \Gamma \times \Gamma \rightharpoonup Q \times \Gamma \times \Gamma \times \{L, R\}$$

TMs with Storage – Example

$$L(M) = \{\mathtt{ab}^n \text{ or } \mathtt{ba}^n \mid n \geq 0\}$$

The following TM with storage accepts $L(M)$, and see the example for $\mathtt{abb} \in L(M)$.[1]



[1] https://plrg.korea.ac.kr/courses/cose215/materials/tm-storage-abn-or-ban.pdf

TMs with Storage are Equivalent to TMs

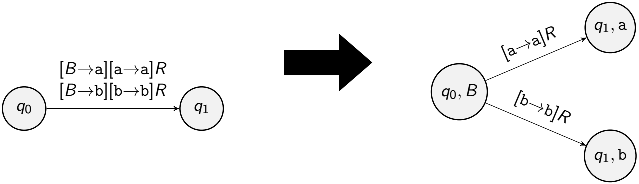
Theorem

A language accepted by a **TM with storage** *is* **recursively enumerable** *(i.e., accepted by a standard* **TM***).*

Proof) We can define an equivalent standard TM by using pairs of states and symbols in the storage as its states:

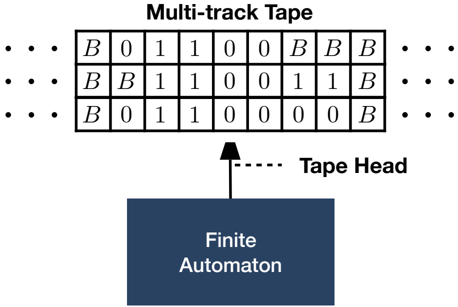
$$\delta'((q, a), b) = \delta(q, a, b)$$

where $Q' = Q \times \Gamma$ and $\delta' : Q' \times \Gamma \rightharpoonup Q' \times \Gamma \times \{L, R\}$.
For example,



Multi-track TMs

We can define a TM with a **multi-track tape**:



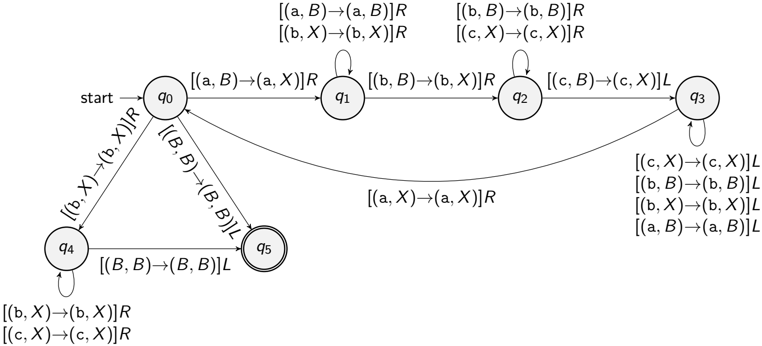
It has a tape with n **tracks** and a **single tape head**:

$$\delta : Q \times \Gamma^n \rightharpoonup Q \times \Gamma^n \times \{L, R\}$$

Multi-track TMs – Example

$$L(M) = \{\mathtt{a}^n\mathtt{b}^n\mathtt{c}^n \mid n \geq 0\}$$

The following multi-track TM accepts $L(M)$, and see the example for $\mathtt{aabbcc} \in L(M)$.[2]



[2] https://plrg.korea.ac.kr/courses/cose215/materials/tm-multi-track-an-bn-cn.pdf

Multi-track TMs are Equivalent to Standard TMs

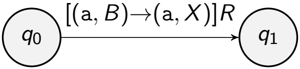
Theorem

A language accepted by a **multi-track TM** *is* **recursively enumerable** *(i.e., accepted by a standard* **TM***).*

Proof) We can define an equivalent standard TM by using n-tuples of symbols as a single symbol:

$$\delta'(q, \alpha) = \delta(q, \alpha)$$

where $\Gamma' = \Gamma^n$ and $\delta' : Q \times \Gamma' \rightharpoonup Q \times \Gamma' \times \{L, R\}$.
For example,

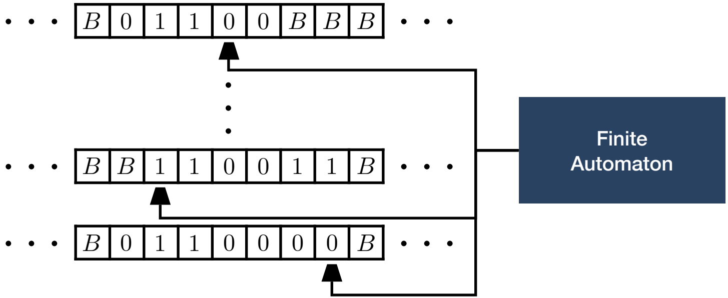


$\cdots$	B	a	b	B	$\cdots$
$\cdots$	B	X	B	B	$\cdots$

$\cdots$	(B, B)	(a, X)	(b, B)	(B, B)	$\cdots$

Multi-tape TMs

We can define a TM with **multiple tapes**:

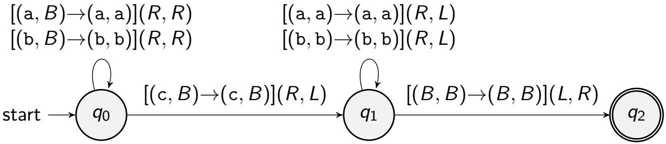


It has *n* **tapes**, and each tape has its **own head** that can move independently:

$$\delta : Q \times \Gamma^n \rightharpoonup Q \times (\Gamma \times \{L, R\})^n$$

$$L(M) = \{w\texttt{c}w^R \mid w \in \{\texttt{a}, \texttt{b}\}^*\}$$

The following multi-tape TM accepts $L(M)$, and see the example for $\texttt{abbcbba} \in L(M)$.[3]



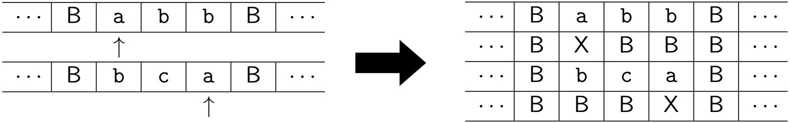
[3] https://plrg.korea.ac.kr/courses/cose215/materials/tm-multi-tape-w-c-wr.pdf

Multi-tape TMs are Equivalent to Standard TMs

Theorem

A language accepted by a **multi-tape TM** *is* **recursively enumerable** *(i.e., accepted by a standard* **TM***).*

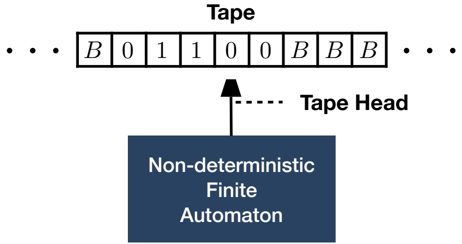
Proof) For a given n-tape TM, we can define an equivalent $2n$-track TM with storage by using **odd** tracks for the original **tapes** and **even** tracks for the **tape heads**:



We can simulate one-step in the n-tape TM by gathering all the symbols pointed by the n heads into the storage, and then taking the action same as done by the n-tape TM.

Non-deterministic TMs

We can define a TM with **non-deterministic transitions**:



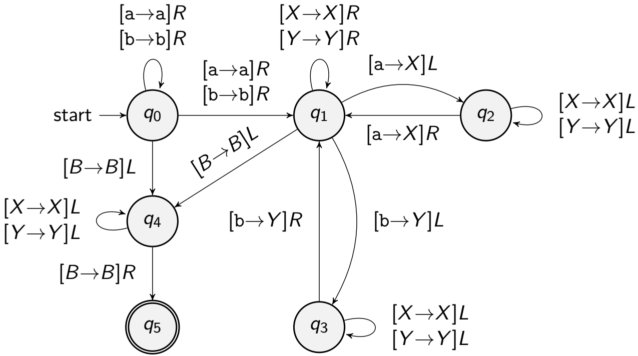
It has a **non-deterministic transition function**:

$$\delta : Q \times \Gamma \to \mathcal{P}(Q \times \Gamma \times \{L, R\})$$

Non-deterministic TMs – Example

$$L(M) = \{ww^R \mid w \in \{\texttt{a}, \texttt{b}\}^*\}$$

The following multi-tape TM accepts $L(M)$, and see the example for abba $\in L(M)$.[4]



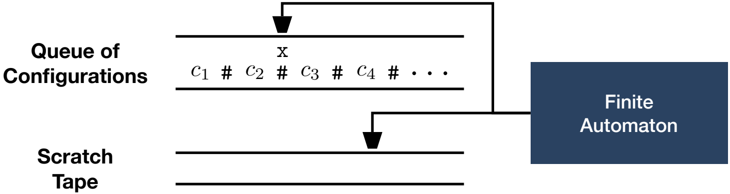
[4] https://plrg.korea.ac.kr/courses/cose215/materials/ntm-w-wr.pdf

Non-deterministic TMs are Equivalent to Standard TMs

Theorem

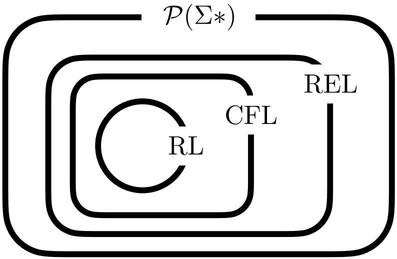
A language accepted by a **non-deterministic TM** *is* **recursively enumerable** *(i.e., accepted by a standard* **TM***).*

Proof) For a given non-deterministic TM, we can define an equivalent 2-tape TM: 1) a 2-track tape to maintain a **queue of configurations** and 2) a normal track to **simulate** the tape of the original TM.



More Extensions of TMs

- There are more extensions of TMs:
  - TMs with **Stay Option** – L: Left, R: Right, and S: **Stay**
  - **Queue Automata** – Automata with **Queue**
  - **Random Access Machines** – TMs with **Random Access Memory**
  - . . .
- They are all **equivalent** to TMs.
- A standard **TM** is the **most powerful model of computation**.



TM (Turing Machine) = ETM (All Extensions of TMs)

TM = REL (Recursively Enumerable Language)

Summary

1. Extensions of Turing Machines
    - TMs with Storage
    - Multi-track TMs
    - Multi-tape TMs
    - Non-deterministic TMs
    - More Extensions of TMs

Next Lecture

- The Origin of Computer Science

Jihyeok Park
jihyeok_park@korea.ac.kr
https://plrg.korea.ac.kr