

Lecture 27 – Course Review

COSE215: Theory of Computation

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2023 Spring

- What is the *mathematical model* of computers?

- What is the *mathematical model* of computers?

Turing Machine!

Let's learn **Turing Machine**

- What is the *mathematical model* of computers?

Turing Machine!

Let's learn **Turing Machine**

- Is it possible to solve *every problem* using computers?

- What is the *mathematical model* of computers?

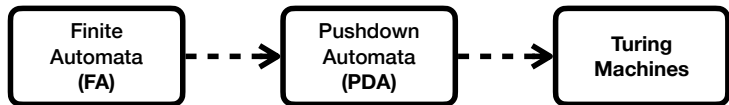
Turing Machine!

Let's learn **Turing Machine**

- Is it possible to solve *every problem* using computers?

No!

Let's learn **Undecidability** and **Intractability**



- **Finite Automata (FA)**
 - Regular Expressions and Languages
 - Applications: text search, etc.
- **Pushdown Automata (PDA)**
 - Context-Free Grammars (CFGs) and Languages (CFLs)
 - Applications: programming languages, natural language processing, etc.
- **Turing Machines (TMs)**
 - Extensions of Turing Machines
 - Undecidability and Intractability

	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata			
(Part 1) Finite Automata			
(Part 0) Basic Concepts			

	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata			
(Part 1) Finite Automata			
(Part 0) Basic Concepts	(Lecture 1) Mathematical Preliminaries		

	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata			
(Part 1) Finite Automata			
(Part 0) Basic Concepts	(Lecture 1) Mathematical Preliminaries	(Lecture 2) Scala	

	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata			
(Part 1) Finite Automata	(Lecture 3) DFA		(Lecture 3) RL
(Part 0) Basic Concepts	(Lecture 1) Mathematical Preliminaries	(Lecture 2) Scala	

	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata			
(Part 1) Finite Automata	(Lecture 4) NFA \rightleftarrows (Lecture 3) DFA		(Lecture 3) RL
(Part 0) Basic Concepts	(Lecture 1) Mathematical Preliminaries	(Lecture 2) Scala	

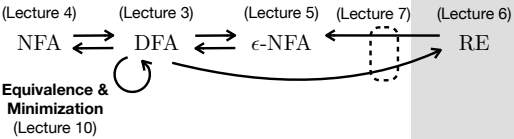
	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata			
(Part 1) Finite Automata	(Lecture 4) (Lecture 3) (Lecture 5) NFA \rightleftarrows DFA \rightleftarrows ϵ -NFA		(Lecture 3) RL
(Part 0) Basic Concepts	(Lecture 1) Mathematical Preliminaries	(Lecture 2) Scala	

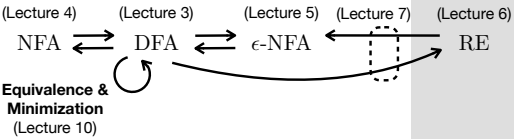
	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata			
(Part 1) Finite Automata	(Lecture 4) (Lecture 3) (Lecture 5) NFA \rightleftarrows DFA \rightleftarrows ϵ -NFA	(Lecture 6) RE	(Lecture 3) RL
(Part 0) Basic Concepts	(Lecture 1) Mathematical Preliminaries	(Lecture 2) Scala	

	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata			
(Part 1) Finite Automata	(Lecture 4) NFA	(Lecture 3) DFA	(Lecture 3) RL
	\rightleftarrows	\rightleftarrows	\leftarrow
		(Lecture 5) ϵ -NFA	(Lecture 6) RE
		\rightleftarrows	\leftarrow
			\leftarrow
(Part 0) Basic Concepts	(Lecture 1) Mathematical Preliminaries		(Lecture 2) Scala

	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata			
(Part 1) Finite Automata	(Lecture 4) NFA	(Lecture 3) DFA	(Lecture 5) ϵ -NFA
	\longleftrightarrow	\longleftrightarrow	\longleftrightarrow
			(Lecture 7) RE
			(Lecture 6)
			(Lecture 3) RL : Closure Properties (Lecture 8)
(Part 0) Basic Concepts	(Lecture 1) Mathematical Preliminaries	(Lecture 2) Scala	

	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata			
(Part 1) Finite Automata	(Lecture 4) NFA	(Lecture 3) DFA	(Lecture 5) ϵ -NFA
	\longleftrightarrow	\longleftrightarrow	\longleftrightarrow
			(Lecture 6) RE
			(Lecture 7)
			(Lecture 3) RL
			Closure Properties (Lecture 8)
			Pumping Lemma (Lecture 9)
(Part 0) Basic Concepts	(Lecture 1) Mathematical Preliminaries	(Lecture 2) Scala	

	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata			
(Part 1) Finite Automata	<p>(Lecture 4) NFA \rightleftarrows DFA \rightleftarrows ϵ-NFA (Lecture 3)</p> <p>Equivalence & Minimization (Lecture 10)</p> 	<p>(Lecture 6) RE</p> <p>(Lecture 3) RL</p> <p>Closure Properties (Lecture 8)</p> <p>Pumping Lemma (Lecture 9)</p>	
(Part 0) Basic Concepts	(Lecture 1) Mathematical Preliminaries	(Lecture 2) Scala	

	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata		(Lecture 11/12) CFG	(Lecture 11) CFL
(Part 1) Finite Automata	(Lecture 4) NFA \rightleftarrows DFA \rightleftarrows ϵ -NFA (Lecture 3) \rightleftarrows RE (Lecture 6) (Lecture 5) \leftarrow (Lecture 7) Equivalence & Minimization (Lecture 10) 		(Lecture 3) RL Closure Properties (Lecture 8) Pumping Lemma (Lecture 9)
(Part 0) Basic Concepts	(Lecture 1) Mathematical Preliminaries	(Lecture 2) Scala	

	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata		(Lecture 11/12) CFG	(Lecture 11) CFL ... (Lecture 13) Parse Trees & Ambiguity
(Part 1) Finite Automata	(Lecture 4) NFA \rightleftarrows (Lecture 3) DFA \rightleftarrows (Lecture 5) ϵ -NFA \leftarrow (Lecture 7) RE \rightarrow (Lecture 6) RE Equivalence & Minimization (Lecture 10)		(Lecture 3) RL Closure Properties (Lecture 8) ... Pumping Lemma (Lecture 9)
(Part 0) Basic Concepts	(Lecture 1) Mathematical Preliminaries	(Lecture 2) Scala	

	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata	(Lecture 14/15) PDA _{FS} PDA _{ES}	(Lecture 11/12) CFG	(Lecture 11) CFL ... (Lecture 13) Parse Trees & Ambiguity
(Part 1) Finite Automata	(Lecture 4) NFA ↔ (Lecture 3) DFA ↔ (Lecture 5) ϵ -NFA Equivalence & Minimization (Lecture 10)	(Lecture 7) RE (Lecture 6)	(Lecture 3) RL Closure Properties (Lecture 8) Pumping Lemma (Lecture 9)
(Part 0) Basic Concepts	(Lecture 1) Mathematical Preliminaries	(Lecture 2) Scala	

	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata	(Lecture 14/15) $PDA_{FS} \rightleftarrows PDA_{ES}$	(Lecture 16) \rightleftarrows	(Lecture 11/12) CFG (Lecture 11) CFL ... (Lecture 13) Parse Trees & Ambiguity
(Part 1) Finite Automata	(Lecture 4) NFA \rightleftarrows DFA (Lecture 3) DFA \rightleftarrows ϵ -NFA (Lecture 5) ϵ -NFA \rightleftarrows RE (Lecture 7) RE \rightleftarrows RE (Lecture 6) Equivalence & Minimization (Lecture 10)		(Lecture 3) RL Closure Properties (Lecture 8) Pumping Lemma (Lecture 9)
(Part 0) Basic Concepts	(Lecture 1) Mathematical Preliminaries	(Lecture 2) Scala	

	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata	<p>(Lecture 14/15)</p> $PDA_{FS} \begin{matrix} \xrightarrow{\text{dashed}} \\ \xleftrightarrow{\text{dashed}} \\ \xleftarrow{\text{dashed}} \end{matrix} PDA_{ES}$ <p>\cup</p> $DPDA_{FS} \supset DPDA_{ES}$ <p>\cup (Lecture 17) $\not\subset$</p>	<p>(Lecture 16)</p> $PDA_{ES} \begin{matrix} \xleftrightarrow{\text{dashed}} \\ \xleftrightarrow{\text{dashed}} \end{matrix} CFG$	<p>(Lecture 11/12)</p> <p>(Lecture 11) CFL ... (Lecture 13) Parse Trees & Ambiguity</p>
(Part 1) Finite Automata	<p>(Lecture 4)</p> $NFA \begin{matrix} \xleftrightarrow{\text{dashed}} \\ \xleftrightarrow{\text{dashed}} \end{matrix} DFA$ <p>Equivalence & Minimization (Lecture 10)</p>	<p>(Lecture 3)</p> $DFA \begin{matrix} \xleftrightarrow{\text{dashed}} \\ \xleftrightarrow{\text{dashed}} \end{matrix} \epsilon\text{-NFA}$ <p>(Lecture 5)</p> $\epsilon\text{-NFA} \begin{matrix} \xleftrightarrow{\text{dashed}} \\ \xleftrightarrow{\text{dashed}} \end{matrix} RE$ <p>(Lecture 7)</p> <p>(Lecture 6)</p>	<p>(Lecture 3)</p> RL <p>⋮</p> <p>Closure Properties (Lecture 8) Pumping Lemma (Lecture 9)</p>
(Part 0) Basic Concepts	<p>(Lecture 1)</p> <p>Mathematical Preliminaries</p>	<p>(Lecture 2)</p> <p>Scala</p>	

	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata	<p>(Lecture 14/15)</p> $\text{PDA}_{\text{FS}} \begin{array}{c} \xrightarrow{\text{---}} \\ \xleftarrow{\text{---}} \end{array} \text{PDA}_{\text{ES}}$ <p>\cup</p> $\text{DPDA}_{\text{FS}} \supset \text{DPDA}_{\text{ES}}$ <p>\cup (Lecture 17) \curvearrowright</p>	<p>(Lecture 16)</p> $\text{PDA}_{\text{ES}} \begin{array}{c} \xrightarrow{\text{---}} \\ \xleftarrow{\text{---}} \end{array} \text{CFG}$ <p>\vdots</p> <p>Chomsky Normal Form (Lecture 18)</p>	<p>(Lecture 11) CFL ... (Lecture 13) Parse Trees & Ambiguity</p>
(Part 1) Finite Automata	<p>(Lecture 4) $\text{NFA} \begin{array}{c} \xrightarrow{\text{---}} \\ \xleftarrow{\text{---}} \end{array} \text{DFA}$</p> <p>(Lecture 3) $\text{DFA} \begin{array}{c} \xrightarrow{\text{---}} \\ \xleftarrow{\text{---}} \end{array} \epsilon\text{-NFA}$</p> <p>(Lecture 5) $\epsilon\text{-NFA} \begin{array}{c} \xrightarrow{\text{---}} \\ \xleftarrow{\text{---}} \end{array} \text{RE}$</p> <p>(Lecture 7) $\text{RE} \begin{array}{c} \xrightarrow{\text{---}} \\ \xleftarrow{\text{---}} \end{array} \text{DFA}$</p> <p>(Lecture 6)</p> <p>Equivalence & Minimization (Lecture 10)</p>		<p>(Lecture 3) RL</p> <p>\vdots</p> <p>Closure Properties (Lecture 8) Pumping Lemma (Lecture 9)</p>
(Part 0) Basic Concepts	<p>(Lecture 1)</p> <p>Mathematical Preliminaries</p>	<p>(Lecture 2)</p> <p>Scala</p>	

	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata	<p>(Lecture 14/15)</p> $\text{PDA}_{\text{FS}} \begin{matrix} \xrightarrow{\text{dashed}} \\ \xleftarrow{\text{dashed}} \end{matrix} \text{PDA}_{\text{ES}} \begin{matrix} \xleftarrow{\text{dashed}} \\ \xrightarrow{\text{dashed}} \end{matrix} \text{CFG}$ <p>(Lecture 16)</p> $\text{DPDA}_{\text{FS}} \supset \text{DPDA}_{\text{ES}}$ <p>(Lecture 17)</p>	<p>(Lecture 11/12)</p> <p>CFG</p> <p>⋮</p> <p>Chomsky Normal Form (Lecture 18)</p>	<p>(Lecture 11)</p> <p>CFL ... Parse Trees & Ambiguity</p> <p>⋮</p> <p>Closure Properties (Lecture 19)</p>
(Part 1) Finite Automata	<p>(Lecture 4)</p> <p>NFA $\xleftrightarrow{\text{dashed}}$ DFA $\xleftrightarrow{\text{dashed}}$ ϵ-NFA $\xleftrightarrow{\text{dashed}}$ RE</p> <p>(Lecture 3)</p> <p>(Lecture 5)</p> <p>(Lecture 7)</p> <p>(Lecture 6)</p> <p>Equivalence & Minimization (Lecture 10)</p>		<p>(Lecture 3)</p> <p>RL</p> <p>⋮</p> <p>Closure Properties (Lecture 8)</p> <p>Pumping Lemma (Lecture 9)</p>
(Part 0) Basic Concepts	<p>(Lecture 1)</p> <p>Mathematical Preliminaries</p>	<p>(Lecture 2)</p> <p>Scala</p>	

	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata	<p>(Lecture 14/15)</p> $\text{PDA}_{\text{FS}} \begin{matrix} \xrightarrow{\text{dashed}} \\ \xleftarrow{\text{dashed}} \end{matrix} \text{PDA}_{\text{ES}} \begin{matrix} \xleftarrow{\text{dashed}} \\ \xrightarrow{\text{dashed}} \end{matrix} \text{CFG}$ <p>(Lecture 16)</p> $\text{DPDA}_{\text{FS}} \supset \text{DPDA}_{\text{ES}}$ <p>(Lecture 17)</p>	<p>(Lecture 11/12)</p> <p>CFG</p> <p>Chomsky Normal Form (Lecture 18)</p>	<p>(Lecture 11)</p> <p>CFL</p> <p>Closure Properties (Lecture 19)</p> <p>(Lecture 13)</p> <p>Parse Trees & Ambiguity</p> <p>Pumping Lemma (Lecture 20)</p>
(Part 1) Finite Automata	<p>(Lecture 4)</p> <p>NFA</p> <p>Equivalence & Minimization (Lecture 10)</p>	<p>(Lecture 3)</p> <p>DFA</p> <p>(Lecture 5)</p> <p>ϵ-NFA</p>	<p>(Lecture 6)</p> <p>RE</p> <p>(Lecture 3)</p> <p>RL</p> <p>Closure Properties (Lecture 8)</p> <p>Pumping Lemma (Lecture 9)</p>
(Part 0) Basic Concepts	<p>(Lecture 1)</p> <p>Mathematical Preliminaries</p>	<p>(Lecture 2)</p> <p>Scala</p>	

	Automata	Grammars	Languages
(Part 3) Turing Machines	(Lecture 21/22) TM		(Lecture 21) REL
(Part 2) Pushdown Automata	(Lecture 14/15) $PDA_{FS} \leftrightarrow PDA_{ES}$ \cup $DPDA_{FS} \supset DPDA_{ES}$ \cup (Lecture 17)	(Lecture 16) \leftrightarrow (Lecture 11/12) CFG ⋮ Chomsky Normal Form (Lecture 18)	(Lecture 11) CFL ⋮ Closure Properties (Lecture 19) (Lecture 13) Parse Trees & Ambiguity ⋮ Pumping Lemma (Lecture 20)
(Part 1) Finite Automata	(Lecture 4) NFA \leftrightarrow (Lecture 3) DFA \leftrightarrow (Lecture 5) ϵ -NFA \leftrightarrow (Lecture 7) RE Equivalence & Minimization (Lecture 10)	(Lecture 6)	(Lecture 3) RL ⋮ Closure Properties (Lecture 8) Pumping Lemma (Lecture 9)
(Part 0) Basic Concepts	(Lecture 1) Mathematical Preliminaries	(Lecture 2) Scala	

	Automata	Grammars	Languages
(Part 3) Turing Machines	(Lecture 23) ETM \longleftrightarrow (Lecture 21/22) TM		(Lecture 21) REL
(Part 2) Pushdown Automata	(Lecture 14/15) PDA _{FS} \longleftrightarrow PDA _{ES} \cup DPDA _{FS} \supset DPDA _{ES} \cup (Lecture 17) \curvearrowright	(Lecture 16) \longleftrightarrow (Lecture 11/12) CFG ⋮ Chomsky Normal Form (Lecture 18)	(Lecture 11) CFL ⋮ Closure Properties (Lecture 19) (Lecture 13) Parse Trees & Ambiguity ⋮ Pumping Lemma (Lecture 20)
(Part 1) Finite Automata	(Lecture 4) NFA \longleftrightarrow (Lecture 3) DFA \longleftrightarrow (Lecture 5) ϵ -NFA \longleftrightarrow (Lecture 7) RE Equivalence & Minimization (Lecture 10)	(Lecture 6)	(Lecture 3) RL ⋮ Closure Properties (Lecture 8) (Lecture 9) Pumping Lemma
(Part 0) Basic Concepts	(Lecture 1) Mathematical Preliminaries	(Lecture 2) Scala	

	Automata	Grammars	Languages
(Part 3) Turing Machines	(Lecture 23) $ETM \rightleftharpoons TM$	(Lecture 21/22) $TM \rightleftharpoons LC$ (Lecture 24)	(Lecture 21) REL
(Part 2) Pushdown Automata	(Lecture 14/15) $PDA_{FS} \rightleftharpoons PDA_{ES}$ \cup $DPDA_{FS} \supset DPDA_{ES}$ \cup (Lecture 17) \curvearrowright	(Lecture 16) $PDA_{ES} \rightleftharpoons CFG$ (Lecture 11/12) Chomsky Normal Form (Lecture 18)	(Lecture 11) CFL (Lecture 13) Parse Trees & Ambiguity (Lecture 19) Closure Properties (Lecture 20) Pumping Lemma
(Part 1) Finite Automata	(Lecture 4) $NFA \rightleftharpoons DFA$ (Lecture 3) $DFA \rightleftharpoons \epsilon\text{-NFA}$ (Lecture 5) $\epsilon\text{-NFA} \rightleftharpoons RE$ (Lecture 7) $RE \rightleftharpoons NFA$ (Lecture 6) Equivalence & Minimization (Lecture 10)		(Lecture 3) RL (Lecture 8) Closure Properties (Lecture 9) Pumping Lemma
(Part 0) Basic Concepts	(Lecture 1) Mathematical Preliminaries	(Lecture 2) Scala	

	Automata	Grammars	Languages
(Part 3) Turing Machines	(Lecture 23) $\text{ETM} \rightleftharpoons \text{TM}$ (Lecture 21/22)	(Lecture 24) $\text{TM} \rightleftharpoons \text{LC}$	(Lecture 21) REL \cup DL (Lecture 25)
(Part 2) Pushdown Automata	(Lecture 14/15) $\text{PDA}_{\text{FS}} \rightleftharpoons \text{PDA}_{\text{ES}}$ \cup $\text{DPDA}_{\text{FS}} \supset \text{DPDA}_{\text{ES}}$ \cup (Lecture 17) \curvearrowright	(Lecture 16) $\text{PDA}_{\text{ES}} \rightleftharpoons \text{CFG}$ \vdots Chomsky Normal Form (Lecture 18)	(Lecture 11) CFL (Lecture 13) Parse Trees & Ambiguity \vdots Closure Properties (Lecture 19) Pumping Lemma (Lecture 20)
(Part 1) Finite Automata	(Lecture 4) $\text{NFA} \rightleftharpoons \text{DFA}$ (Lecture 3) (Lecture 5) $\text{DFA} \rightleftharpoons \epsilon\text{-NFA}$ (Lecture 7) (Lecture 6) $\epsilon\text{-NFA} \rightleftharpoons \text{RE}$ (Lecture 10) Equivalence & Minimization		(Lecture 3) RL \vdots Closure Properties (Lecture 8) Pumping Lemma (Lecture 9)
(Part 0) Basic Concepts	(Lecture 1) Mathematical Preliminaries	(Lecture 2) Scala	

	Automata	Grammars	Languages
(Part 3) Turing Machines	(Lecture 23) $\text{ETM} \rightleftharpoons$ (Lecture 21/22) $\text{TM} \rightleftharpoons$ (Lecture 24) LC		(Lecture 21) REL (Lecture 26) \cup $\text{DL} \supset \text{NP} \stackrel{?}{=} \text{P}$ (Lecture 25) \supset
(Part 2) Pushdown Automata	(Lecture 14/15) $\text{PDA}_{\text{FS}} \rightleftharpoons \text{PDA}_{\text{ES}}$ \cup $\text{DPDA}_{\text{FS}} \supset \text{DPDA}_{\text{ES}}$ \cup (Lecture 17) \cup	(Lecture 16) \rightleftharpoons (Lecture 11/12) CFG \vdots Chomsky Normal Form (Lecture 18)	(Lecture 11) CFL (Lecture 13) Parse Trees & Ambiguity \vdots Closure Properties (Lecture 19) Pumping Lemma (Lecture 20)
(Part 1) Finite Automata	(Lecture 4) $\text{NFA} \rightleftharpoons$ (Lecture 3) $\text{DFA} \rightleftharpoons$ (Lecture 5) $\epsilon\text{-NFA} \rightleftharpoons$ (Lecture 7) RE (Lecture 6) Equivalence & Minimization (Lecture 10)		(Lecture 3) RL \vdots Closure Properties (Lecture 8) Pumping Lemma (Lecture 9)
(Part 0) Basic Concepts	(Lecture 1) Mathematical Preliminaries	(Lecture 2) Scala	

- I hope you enjoyed the class!

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