

# Lecture 27 – Course Review

## COSE215: Theory of Computation

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2024 Spring

- What is the *mathematical model* of computers?

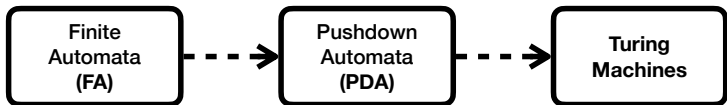
## **Turing Machine!**

Let's learn **Turing Machine**

- Is it possible to solve *every problem* using computers?

**No!**

Let's learn **Undecidability** and **Intractability**



- **Finite Automata (FA)**

- Regular Expressions and Languages
- Applications: text search, etc.

- **Pushdown Automata (PDA)**

- Context-Free Grammars (CFGs) and Languages (CFLs)
- Applications: programming languages, natural language processing, etc.

- **Turing Machines (TMs)**

- Extensions of Turing Machines
- Undecidability and Intractability

|   | Automata | Grammars | Languages |
|---|----------|----------|-----------|
| <b>(Part 3)<br/>Turing<br/>Machines</b>   |          |          |           |
| <b>(Part 2)<br/>Pushdown<br/>Automata</b> |          |          |           |
| <b>(Part 1)<br/>Finite<br/>Automata</b>   |          |          |           |
| <b>(Part 0)<br/>Basic<br/>Concepts</b>    |          |          |           |

|   | Automata   | Grammars | Languages |
|---|--|----------|-----------|
| <b>(Part 3)<br/>Turing<br/>Machines</b>   |  |          |           |
| <b>(Part 2)<br/>Pushdown<br/>Automata</b> |  |          |           |
| <b>(Part 1)<br/>Finite<br/>Automata</b>   |  |          |           |
| <b>(Part 0)<br/>Basic<br/>Concepts</b>    | (Lecture 1)<br><b>Mathematical<br/>Preliminaries</b> |          |           |

|                                  | Automata   | Grammars                    | Languages |
|----------------------------------|--|-----------------------------|-----------|
| (Part 3)<br>Turing<br>Machines   |  |                             |           |
| (Part 2)<br>Pushdown<br>Automata |  |                             |           |
| (Part 1)<br>Finite<br>Automata   |  |                             |           |
| (Part 0)<br>Basic<br>Concepts    | (Lecture 1)<br><b>Mathematical<br/>Preliminaries</b> | (Lecture 2)<br><b>Scala</b> |           |

|   | Automata   | Grammars                    | Languages         |
|---|--|-----------------------------|-------------------|
| <b>(Part 3)<br/>Turing<br/>Machines</b>   |  |                             |                   |
| <b>(Part 2)<br/>Pushdown<br/>Automata</b> |  |                             |                   |
| <b>(Part 1)<br/>Finite<br/>Automata</b>   | (Lecture 3)<br>DFA                                   |                             | (Lecture 3)<br>RL |
| <b>(Part 0)<br/>Basic<br/>Concepts</b>    | (Lecture 1)<br><b>Mathematical<br/>Preliminaries</b> | (Lecture 2)<br><b>Scala</b> |                   |

|   | Automata  | Grammars                    | Languages         |
|---|---|-----------------------------|-------------------|
| <b>(Part 3)<br/>Turing<br/>Machines</b>   |   |                             |                   |
| <b>(Part 2)<br/>Pushdown<br/>Automata</b> |   |                             |                   |
| <b>(Part 1)<br/>Finite<br/>Automata</b>   | (Lecture 4)<br>NFA $\longleftrightarrow$ (Lecture 3)<br>DFA |                             | (Lecture 3)<br>RL |
| <b>(Part 0)<br/>Basic<br/>Concepts</b>    | (Lecture 1)<br><b>Mathematical<br/>Preliminaries</b>        | (Lecture 2)<br><b>Scala</b> |                   |



|                                  | Automata  | Grammars             | Languages         |
|----------------------------------|---|----------------------|-------------------|
| (Part 3)<br>Turing<br>Machines   |   |                      |                   |
| (Part 2)<br>Pushdown<br>Automata |   |                      |                   |
| (Part 1)<br>Finite<br>Automata   | (Lecture 4) NFA $\longleftrightarrow$ (Lecture 3) DFA $\longleftrightarrow$ (Lecture 5) $\epsilon$ -NFA |                      | (Lecture 3)<br>RL |
| (Part 0)<br>Basic<br>Concepts    | (Lecture 1)<br>Mathematical<br>Preliminaries  | (Lecture 2)<br>Scala |                   |

|   | Automata  | Grammars                    | Languages      |
|---|---|-----------------------------|----------------|
| <b>(Part 3)</b><br><b>Turing Machines</b>   |   |                             |                |
| <b>(Part 2)</b><br><b>Pushdown Automata</b> |   |                             |                |
| <b>(Part 1)</b><br><b>Finite Automata</b>   | (Lecture 4) NFA $\longleftrightarrow$ (Lecture 3) DFA $\longleftrightarrow$ (Lecture 5) $\epsilon$ -NFA | (Lecture 6) RE              | (Lecture 3) RL |
| <b>(Part 0)</b><br><b>Basic Concepts</b>    | (Lecture 1)<br><b>Mathematical Preliminaries</b>  | (Lecture 2)<br><b>Scala</b> |                |

|                                  | Automata                                     | Grammars                       | Languages         |
|----------------------------------|--|--------------------------------|-------------------|
| (Part 3)<br>Turing<br>Machines   |  |                                |                   |
| (Part 2)<br>Pushdown<br>Automata |  |                                |                   |
| (Part 1)<br>Finite<br>Automata   | (Lecture 4)<br>NFA                           | (Lecture 3)<br>DFA             | (Lecture 3)<br>RL |
|                                  | $\rightleftarrows$                           | $\rightleftarrows$             | $\leftarrow$      |
|                                  |  | (Lecture 5)<br>$\epsilon$ -NFA | (Lecture 6)<br>RE |
|                                  |  | $\leftarrow$                   | $\rightarrow$     |
|                                  |  | (Lecture 7)                    |                   |
| (Part 0)<br>Basic<br>Concepts    | (Lecture 1)<br>Mathematical<br>Preliminaries | (Lecture 2)<br>Scala           |                   |

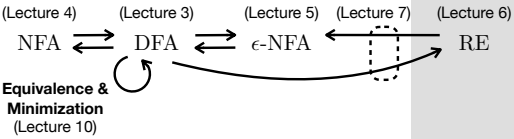
|                                  | Automata             | Grammars           | Languages  |
|----------------------------------|----------------------|--------------------|--|
| (Part 3)<br>Turing<br>Machines   |                      |                    |  |
| (Part 2)<br>Pushdown<br>Automata |                      |                    |  |
| (Part 1)<br>Finite<br>Automata   | (Lecture 4)<br>NFA   | (Lecture 3)<br>DFA | (Lecture 3)<br>RL<br>⋮<br>Closure<br>Properties<br>(Lecture 8) |
| (Part 0)<br>Basic<br>Concepts    | (Lecture 5)<br>ε-NFA | (Lecture 6)<br>RE  |  |

Diagram illustrating the relationships between Automata, Grammars, and Languages across different parts of the course.

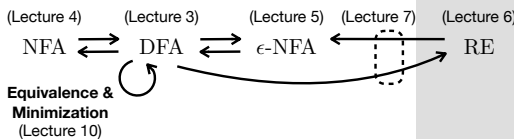
In the (Part 1) Finite Automata row, the following relationships are shown:

- NFA (Lecture 4) and DFA (Lecture 3) are connected by a double-headed arrow.
- DFA (Lecture 3) and ε-NFA (Lecture 5) are connected by a double-headed arrow.
- ε-NFA (Lecture 5) and RE (Lecture 6) are connected by a double-headed arrow.
- RE (Lecture 6) and RL (Lecture 3) are connected by a double-headed arrow.
- A dashed box encloses the ε-NFA and RE nodes.
- A curved arrow points from the ε-NFA node to the RE node.

|                                  | Automata                                     | Grammars             | Languages   |
|----------------------------------|--|----------------------|---|
| (Part 3)<br>Turing<br>Machines   |  |                      |   |
| (Part 2)<br>Pushdown<br>Automata |  |                      |   |
| (Part 1)<br>Finite<br>Automata   | (Lecture 4)<br>NFA                           | (Lecture 3)<br>DFA   | (Lecture 5)<br>$\epsilon$ -NFA  |
|                                  | ↔  | ↔                    | ←   |
|                                  |  |                      | (Lecture 7) <div style="border: 1px dashed black; border-radius: 50%; width: 20px; height: 20px; margin: 5px auto;"></div>  |
|                                  |  |                      | (Lecture 6)<br>RE   |
|                                  |  |                      | (Lecture 3)<br>RL <div style="display: flex; justify-content: space-around; margin-top: 10px;"> <div style="text-align: center;">             Closure<br/>Properties<br/>(Lecture 8)           </div> <div style="text-align: center;">             Pumping<br/>Lemma<br/>(Lecture 9)           </div> </div> |
| (Part 0)<br>Basic<br>Concepts    | (Lecture 1)<br>Mathematical<br>Preliminaries | (Lecture 2)<br>Scala |   |

|   | Automata  | Grammars   | Languages |
|---|---|--|-----------|
| <b>(Part 3)</b><br>Turing<br>Machines   |   |  |           |
| <b>(Part 2)</b><br>Pushdown<br>Automata |   |  |           |
| <b>(Part 1)</b><br>Finite<br>Automata   | <p>(Lecture 4) NFA <math>\rightleftarrows</math> DFA <math>\rightleftarrows</math> <math>\epsilon</math>-NFA (Lecture 3)</p> <p>Equivalence &amp; Minimization (Lecture 10)</p>  | <p>(Lecture 6) RE</p> <p>(Lecture 3) RL</p> <p>Closure Properties (Lecture 8)</p> <p>Pumping Lemma (Lecture 9)</p> |           |
| <b>(Part 0)</b><br>Basic<br>Concepts    | (Lecture 1)<br>Mathematical<br>Preliminaries  | (Lecture 2)<br>Scala   |           |

|   | Automata   | Grammars               | Languages   |
|---|--|------------------------|---|
| <b>(Part 3)</b><br>Turing<br>Machines   |  |                        |   |
| <b>(Part 2)</b><br>Pushdown<br>Automata |  | (Lecture 11/12)<br>CFG | (Lecture 11)<br>CFL   |
| <b>(Part 1)</b><br>Finite<br>Automata   | (Lecture 4) NFA $\rightleftarrows$ DFA $\rightleftarrows$ $\epsilon$ -NFA $\leftarrow$ (Lecture 7) RE<br>(Lecture 3) $\leftarrow$ (Lecture 5)<br>Equivalence & Minimization (Lecture 10) | (Lecture 6)            | (Lecture 3) RL<br>Closure Properties (Lecture 8)    Pumping Lemma (Lecture 9) |
| <b>(Part 0)</b><br>Basic<br>Concepts    | (Lecture 1)<br>Mathematical Preliminaries  | (Lecture 2)<br>Scala   |   |

|   | Automata  | Grammars                    | Languages  |
|---|---|-----------------------------|--|
| <b>(Part 3)</b><br>Turing<br>Machines   |   |                             |  |
| <b>(Part 2)</b><br>Pushdown<br>Automata |   | (Lecture 11/12)<br>CFG      | (Lecture 11) CFL ... (Lecture 13) <b>Parse Trees &amp; Ambiguity</b>           |
| <b>(Part 1)</b><br>Finite<br>Automata   | (Lecture 4) NFA $\rightleftarrows$ (Lecture 3) DFA $\rightleftarrows$ (Lecture 5) $\epsilon$ -NFA $\leftarrow$ (Lecture 7) RE $\rightarrow$ (Lecture 6) RE<br>Equivalence & Minimization (Lecture 10)  |                             | (Lecture 3) RL<br>Closure Properties (Lecture 8) ... Pumping Lemma (Lecture 9) |
| <b>(Part 0)</b><br>Basic<br>Concepts    | (Lecture 1)<br><b>Mathematical Preliminaries</b>  | (Lecture 2)<br><b>Scala</b> |  |



|   | Automata   | Grammars                    | Languages   |
|---|--|-----------------------------|---|
| <b>(Part 3)<br/>Turing<br/>Machines</b>   |  |                             |   |
| <b>(Part 2)<br/>Pushdown<br/>Automata</b> | (Lecture 14/15)<br>PDA <sub>FS</sub> PDA <sub>ES</sub>   | (Lecture 11/12)<br>CFG      | (Lecture 11) CFL ... (Lecture 13) <b>Parse Trees &amp; Ambiguity</b>            |
| <b>(Part 1)<br/>Finite<br/>Automata</b>   | (Lecture 4) NFA ↔ (Lecture 3) DFA ↔ (Lecture 5) $\epsilon$ -NFA<br>Equivalence & Minimization (Lecture 10) | (Lecture 7) RE              | (Lecture 3) RL<br>Closure Properties (Lecture 8)      Pumping Lemma (Lecture 9) |
| <b>(Part 0)<br/>Basic<br/>Concepts</b>    | (Lecture 1)<br><b>Mathematical Preliminaries</b>   | (Lecture 2)<br><b>Scala</b> |   |

|   | Automata  | Grammars  | Languages  |
|---|---|---|--|
| <b>(Part 3)<br/>Turing<br/>Machines</b>   |   |   |  |
| <b>(Part 2)<br/>Pushdown<br/>Automata</b> | (Lecture 14/15)<br>$PDA_{FS} \rightleftarrows PDA_{ES}$   | (Lecture 16)<br>$\rightleftarrows$  | (Lecture 11/12) CFG<br>(Lecture 11) CFL ...<br>(Lecture 13) <b>Parse Trees &amp; Ambiguity</b> |
| <b>(Part 1)<br/>Finite<br/>Automata</b>   | (Lecture 4) NFA $\rightleftarrows$<br>(Lecture 3) DFA<br>(Lecture 5) $\epsilon$ -NFA<br>(Lecture 7) $\rightleftarrows$<br>(Lecture 6) RE<br>Equivalence & Minimization (Lecture 10) | (Lecture 3) RL<br>Closure Properties (Lecture 8)<br>Pumping Lemma (Lecture 9) |  |
| <b>(Part 0)<br/>Basic<br/>Concepts</b>    | (Lecture 1)<br><b>Mathematical Preliminaries</b>  | (Lecture 2)<br><b>Scala</b>   |  |

|   | Automata  | Grammars  | Languages |
|---|---|---|-----------|
| <b>(Part 3)<br/>Turing<br/>Machines</b>   |   |   |           |
| <b>(Part 2)<br/>Pushdown<br/>Automata</b> | <p>(Lecture 14/15)</p> $\text{PDA}_{\text{FS}} \begin{array}{c} \xleftrightarrow{\text{---}} \\ \xleftrightarrow{\text{---}} \\ \xleftrightarrow{\text{---}} \end{array} \text{PDA}_{\text{ES}} \begin{array}{c} \xleftrightarrow{\text{---}} \\ \xleftrightarrow{\text{---}} \\ \xleftrightarrow{\text{---}} \end{array} \text{CFG}$ <p>(Lecture 16)</p> $\text{DPDA}_{\text{FS}} \supset \text{DPDA}_{\text{ES}}$ <p>(Lecture 17)</p>                   | <p>(Lecture 11/12)</p> <p>(Lecture 11) CFL ... (Lecture 13) <b>Parse Trees &amp; Ambiguity</b></p>  |           |
| <b>(Part 1)<br/>Finite<br/>Automata</b>   | <p>(Lecture 4) (Lecture 3) (Lecture 5) (Lecture 7) (Lecture 6)</p> $\text{NFA} \begin{array}{c} \xleftrightarrow{\text{---}} \\ \xleftrightarrow{\text{---}} \end{array} \text{DFA} \begin{array}{c} \xleftrightarrow{\text{---}} \\ \xleftrightarrow{\text{---}} \end{array} \epsilon\text{-NFA} \begin{array}{c} \xleftrightarrow{\text{---}} \\ \xleftrightarrow{\text{---}} \end{array} \text{RE}$ <p>Equivalence &amp; Minimization (Lecture 10)</p> | <p>(Lecture 3)</p> <p>RL</p> <p>Closure Properties (Lecture 8)</p> <p>Pumping Lemma (Lecture 9)</p> |           |
| <b>(Part 0)<br/>Basic<br/>Concepts</b>    | <p>(Lecture 1)</p> <p><b>Mathematical Preliminaries</b></p>   | <p>(Lecture 2)</p> <p><b>Scala</b></p>  |           |

|   | Automata   | Grammars  | Languages   |
|---|--|---|---|
| <b>(Part 3)<br/>Turing<br/>Machines</b>   |  |   |   |
| <b>(Part 2)<br/>Pushdown<br/>Automata</b> | <p>(Lecture 14/15)</p> $\text{PDA}_{\text{FS}} \begin{array}{c} \xleftrightarrow{\text{---}} \\ \xleftrightarrow{\text{---}} \\ \xleftrightarrow{\text{---}} \end{array} \text{PDA}_{\text{ES}} \begin{array}{c} \xleftarrow{\text{---}} \\ \xleftarrow{\text{---}} \\ \xleftarrow{\text{---}} \end{array}$ <p><math>\cup</math></p> $\text{DPDA}_{\text{FS}} \supset \text{DPDA}_{\text{ES}}$ <p><math>\cup</math> (Lecture 17) <math>\cup</math></p>                     | <p>(Lecture 16)</p> $\text{PDA}_{\text{ES}} \begin{array}{c} \xleftrightarrow{\text{---}} \\ \xleftrightarrow{\text{---}} \\ \xleftrightarrow{\text{---}} \end{array} \text{CFG}$ <p>⋮</p> <p><b>Chomsky Normal Form</b><br/>(Lecture 18)</p> | <p>(Lecture 11) CFL ...</p> <p>(Lecture 13) <b>Parse Trees &amp; Ambiguity</b></p>  |
| <b>(Part 1)<br/>Finite<br/>Automata</b>   | <p>(Lecture 4) NFA <math>\xleftrightarrow{\text{---}}</math> DFA</p> <p>(Lecture 3) DFA <math>\xleftrightarrow{\text{---}}</math> <math>\epsilon</math>-NFA</p> <p>(Lecture 5) <math>\epsilon</math>-NFA <math>\xleftrightarrow{\text{---}}</math> RE</p> <p>(Lecture 7) RE <math>\xleftrightarrow{\text{---}}</math> DFA</p> <p>(Lecture 6) RE <math>\xleftrightarrow{\text{---}}</math> <math>\epsilon</math>-NFA</p> <p>Equivalence &amp; Minimization (Lecture 10)</p> |   | <p>(Lecture 3) RL</p> <p>⋮</p> <p><b>Closure Properties</b><br/>(Lecture 8)</p> <p><b>Pumping Lemma</b><br/>(Lecture 9)</p> |
| <b>(Part 0)<br/>Basic<br/>Concepts</b>    | <p>(Lecture 1)</p> <p><b>Mathematical Preliminaries</b></p>  | <p>(Lecture 2)</p> <p><b>Scala</b></p>  |   |

|   | Automata   | Grammars  | Languages   |
|---|--|---|---|
| <b>(Part 3)<br/>Turing<br/>Machines</b>   |  |   |   |
| <b>(Part 2)<br/>Pushdown<br/>Automata</b> | <p>(Lecture 14/15)</p> $\text{PDA}_{\text{FS}} \begin{array}{c} \xrightarrow{\text{---}} \\ \xleftarrow{\text{---}} \end{array} \text{PDA}_{\text{ES}}$ <p><math>\cup</math></p> $\text{DPDA}_{\text{FS}} \supset \text{DPDA}_{\text{ES}}$ <p><math>\cup</math> (Lecture 17) <math>\not\subset</math></p>  | <p>(Lecture 16)</p> $\text{PDA}_{\text{ES}} \begin{array}{c} \xrightarrow{\text{---}} \\ \xleftarrow{\text{---}} \end{array} \text{CFG}$ <p>(Lecture 11/12)</p> $\vdots$ <p><b>Chomsky Normal Form</b><br/>(Lecture 18)</p> | <p>(Lecture 11)</p> $\text{CFL} \dots$ <p>(Lecture 13)</p> <p><b>Parse Trees &amp; Ambiguity</b></p> <p><math>\vdots</math></p> <p><b>Closure Properties</b><br/>(Lecture 19)</p> |
| <b>(Part 1)<br/>Finite<br/>Automata</b>   | <p>(Lecture 4)</p> $\text{NFA} \begin{array}{c} \xrightarrow{\text{---}} \\ \xleftarrow{\text{---}} \end{array} \text{DFA}$ <p>(Lecture 3)</p> $\text{DFA} \begin{array}{c} \xrightarrow{\text{---}} \\ \xleftarrow{\text{---}} \end{array} \epsilon\text{-NFA}$ <p>(Lecture 5)</p> $\epsilon\text{-NFA} \begin{array}{c} \xrightarrow{\text{---}} \\ \xleftarrow{\text{---}} \end{array} \text{RE}$ <p>(Lecture 7)</p> <p>(Lecture 6)</p> <p><b>Equivalence &amp; Minimization</b><br/>(Lecture 10)</p> |   | <p>(Lecture 3)</p> $\text{RL}$ <p><math>\vdots</math></p> <p><b>Closure Properties</b><br/>(Lecture 8)</p> <p><b>Pumping Lemma</b><br/>(Lecture 9)</p>                            |
| <b>(Part 0)<br/>Basic<br/>Concepts</b>    | <p>(Lecture 1)</p> <p><b>Mathematical Preliminaries</b></p>  | <p>(Lecture 2)</p> <p><b>Scala</b></p>  |   |

|                                       | Automata  | Grammars   | Languages   |
|---------------------------------------|---|--|---|
| <b>(Part 3)<br/>Turing Machines</b>   |   |  |   |
| <b>(Part 2)<br/>Pushdown Automata</b> | <p>(Lecture 14/15)</p> $\text{PDA}_{\text{FS}} \begin{array}{c} \xrightarrow{\text{---}} \\ \xleftarrow{\text{---}} \end{array} \text{PDA}_{\text{ES}}$ <p><math>\cup</math></p> $\text{DPDA}_{\text{FS}} \supset \text{DPDA}_{\text{ES}}$ <p><math>\cup</math> (Lecture 17) <math>\not\subseteq</math></p> | <p>(Lecture 16)</p> $\text{PDA}_{\text{ES}} \begin{array}{c} \xrightarrow{\text{---}} \\ \xleftarrow{\text{---}} \end{array} \text{CFG}$ <p>(Lecture 11/12)</p> <p>CFG</p> <p>⋮</p> <p><b>Chomsky Normal Form</b><br/>(Lecture 18)</p> | <p>(Lecture 11)</p> <p>CFL</p> <p>⋮</p> <p><b>Closure Properties</b><br/>(Lecture 19)</p> <p>(Lecture 13)</p> <p><b>Parse Trees &amp; Ambiguity</b></p> <p>⋮</p> <p><b>Pumping Lemma</b><br/>(Lecture 20)</p> |
| <b>(Part 1)<br/>Finite Automata</b>   | <p>(Lecture 4)</p> <p>NFA</p> <p>↔</p> <p>(Lecture 3)</p> <p>DFA</p> <p>↔</p> <p>(Lecture 5)</p> <p><math>\epsilon</math>-NFA</p> <p>↔</p> <p>(Lecture 7)</p> <p>RE</p> <p>(Lecture 6)</p> <p>↔</p> <p>Equivalence &amp; Minimization<br/>(Lecture 10)</p>  |  | <p>(Lecture 3)</p> <p>RL</p> <p>⋮</p> <p><b>Closure Properties</b><br/>(Lecture 8)</p> <p>⋮</p> <p><b>Pumping Lemma</b><br/>(Lecture 9)</p>   |
| <b>(Part 0)<br/>Basic Concepts</b>    | <p>(Lecture 1)</p> <p><b>Mathematical Preliminaries</b></p>   | <p>(Lecture 2)</p> <p><b>Scala</b></p>   |   |

|   | Automata   | Grammars   | Languages  |
|---|--|--|--|
| <b>(Part 3)<br/>Turing<br/>Machines</b>   | (Lecture 21/22)<br>TM  |  | (Lecture 21)<br>REL  |
| <b>(Part 2)<br/>Pushdown<br/>Automata</b> | <p>(Lecture 14/15)</p> $\text{PDA}_{\text{FS}} \begin{array}{c} \xleftrightarrow{\text{dashed}} \\ \xleftrightarrow{\text{solid}} \\ \xleftrightarrow{\text{dashed}} \end{array} \text{PDA}_{\text{ES}} \xleftrightarrow{\text{dashed}} \text{CFG}$ <p><math>\cup</math></p> $\text{DPDA}_{\text{FS}} \supset \text{DPDA}_{\text{ES}}$ <p><math>\cup</math> (Lecture 17) <math>\cup</math></p>   | <p>(Lecture 16)</p> <p>(Lecture 11/12)</p> <p>CFG</p> <p>⋮</p> <p>Chomsky Normal Form (Lecture 18)</p> | <p>(Lecture 11)</p> <p>CFL</p> <p>⋮</p> <p>Closure Properties (Lecture 19)</p> <p>(Lecture 13) Parse Trees &amp; Ambiguity</p> <p>(Lecture 20) Pumping Lemma</p> |
| <b>(Part 1)<br/>Finite<br/>Automata</b>   | <p>(Lecture 4) (Lecture 3) (Lecture 5) (Lecture 7) (Lecture 6)</p> $\text{NFA} \begin{array}{c} \xleftrightarrow{\text{dashed}} \\ \xleftrightarrow{\text{solid}} \end{array} \text{DFA} \begin{array}{c} \xleftrightarrow{\text{dashed}} \\ \xleftrightarrow{\text{solid}} \end{array} \epsilon\text{-NFA} \begin{array}{c} \xleftrightarrow{\text{dashed}} \\ \xleftrightarrow{\text{solid}} \end{array} \text{RE}$ <p>Equivalence &amp; Minimization (Lecture 10)</p> |  | <p>(Lecture 3)</p> <p>RL</p> <p>⋮</p> <p>Closure Properties (Lecture 8)</p> <p>(Lecture 9) Pumping Lemma</p>   |
| <b>(Part 0)<br/>Basic<br/>Concepts</b>    | (Lecture 1)<br>Mathematical Preliminaries  | (Lecture 2)<br>Scala   |  |

|                                       | Automata  | Grammars  | Languages   |
|---------------------------------------|---|---|---|
| <b>(Part 3)<br/>Turing Machines</b>   | (Lecture 23)<br>ETM $\longleftrightarrow$ (Lecture 21/22)<br>TM   |   | (Lecture 21)<br>REL   |
| <b>(Part 2)<br/>Pushdown Automata</b> | (Lecture 14/15)<br>PDA <sub>FS</sub> $\longleftrightarrow$ PDA <sub>ES</sub><br>$\cup$<br>DPDA <sub>FS</sub> $\supset$ DPDA <sub>ES</sub><br>$\cup$ (Lecture 17) $\curvearrowright$     | (Lecture 16)<br>$\longleftrightarrow$<br>(Lecture 11/12)<br>CFG<br>⋮<br>Chomsky Normal Form<br>(Lecture 18) | (Lecture 11) CFL<br>⋮<br>Closure Properties<br>(Lecture 19)<br>(Lecture 13) Parse Trees & Ambiguity<br>⋮<br>Pumping Lemma<br>(Lecture 20) |
| <b>(Part 1)<br/>Finite Automata</b>   | (Lecture 4) NFA $\longleftrightarrow$ (Lecture 3) DFA $\longleftrightarrow$ (Lecture 5) $\epsilon$ -NFA $\longleftrightarrow$ (Lecture 7) RE<br>Equivalence & Minimization (Lecture 10) | (Lecture 6)   | (Lecture 3) RL<br>⋮<br>Closure Properties (Lecture 8)<br>⋮<br>Pumping Lemma (Lecture 9)   |
| <b>(Part 0)<br/>Basic Concepts</b>    | (Lecture 1)<br>Mathematical Preliminaries   | (Lecture 2)<br>Scala  |   |



|                                       | Automata  | Grammars   | Languages   |
|---------------------------------------|---|--|---|
| <b>(Part 3)<br/>Turing Machines</b>   | (Lecture 23)<br>$ETM \rightleftharpoons TM$   | (Lecture 21/22) $TM \rightleftharpoons LC$<br>(Lecture 24)   | (Lecture 21)<br>REL   |
| <b>(Part 2)<br/>Pushdown Automata</b> | (Lecture 14/15)<br>$PDA_{FS} \rightleftharpoons PDA_{ES}$<br>$\cup$<br>$DPDA_{FS} \supset DPDA_{ES}$<br>$\cup$ (Lecture 17) $\curvearrowright$  | (Lecture 16) $PDA_{ES} \rightleftharpoons CFG$<br>(Lecture 11/12)<br>Chomsky Normal Form<br>(Lecture 18) | (Lecture 11) CFL<br>(Lecture 13) Parse Trees & Ambiguity<br>(Lecture 19) Closure Properties<br>(Lecture 20) Pumping Lemma |
| <b>(Part 1)<br/>Finite Automata</b>   | (Lecture 4) $NFA \rightleftharpoons DFA$<br>(Lecture 3) $DFA \rightleftharpoons \epsilon\text{-NFA}$<br>(Lecture 5) $\epsilon\text{-NFA} \rightleftharpoons RE$<br>(Lecture 7) $RE \rightleftharpoons DFA$<br>(Lecture 6) |  | (Lecture 3) RL<br>(Lecture 8) Closure Properties<br>(Lecture 9) Pumping Lemma   |
| <b>(Part 0)<br/>Basic Concepts</b>    | (Lecture 1)<br>Mathematical Preliminaries   | (Lecture 2)<br>Scala   |   |

|                                       | Automata  | Grammars   | Languages   |
|---------------------------------------|---|--|---|
| <b>(Part 3)<br/>Turing Machines</b>   | (Lecture 23)<br>$ETM \rightleftharpoons TM$   | (Lecture 21/22)<br>$TM \rightleftharpoons LC$  | (Lecture 24)<br>REL<br>$\cup$<br>DL<br>(Lecture 25)   |
| <b>(Part 2)<br/>Pushdown Automata</b> | (Lecture 14/15)<br>$PDA_{FS} \rightleftharpoons PDA_{ES}$<br>$\cup$<br>$DPDA_{FS} \supset DPDA_{ES}$<br>$\cup$ (Lecture 17) | (Lecture 16)<br>$PDA_{ES} \rightleftharpoons CFG$<br>$\vdots$<br>Chomsky Normal Form<br>(Lecture 18) | (Lecture 11) CFL<br>$\vdots$<br>Closure Properties<br>(Lecture 19)<br>(Lecture 13) Parse Trees & Ambiguity<br>$\vdots$<br>Pumping Lemma<br>(Lecture 20) |
| <b>(Part 1)<br/>Finite Automata</b>   | (Lecture 4) NFA<br>$\rightleftharpoons$ (Lecture 3) DFA<br>Equivalence & Minimization<br>(Lecture 10)                       | (Lecture 5) $\epsilon$ -NFA<br>$\rightleftharpoons$ (Lecture 7) RE<br>(Lecture 6)                    | (Lecture 3) RL<br>$\vdots$<br>Closure Properties<br>(Lecture 8)<br>Pumping Lemma<br>(Lecture 9)   |
| <b>(Part 0)<br/>Basic Concepts</b>    | (Lecture 1)<br>Mathematical Preliminaries   | (Lecture 2)<br>Scala   |   |

|   | Automata  | Grammars   | Languages   |
|---|---|--|---|
| <b>(Part 3)<br/>Turing<br/>Machines</b>   | (Lecture 23)<br>$ETM \xleftrightarrow{\hspace{1cm}} TM$<br>(Lecture 21/22)  | (Lecture 24)<br>$TM \xleftrightarrow{\hspace{1cm}} LC$   | (Lecture 21)<br>$REL$<br>$\cup$<br>$DL \supset NP \stackrel{?}{=} P$<br>(Lecture 25)                |
| <b>(Part 2)<br/>Pushdown<br/>Automata</b> | (Lecture 14/15)<br>$PDA_{FS} \xleftrightarrow{\hspace{1cm}} PDA_{ES}$<br>$\cup$<br>$DPDA_{FS} \supset DPDA_{ES}$<br>(Lecture 17)  | (Lecture 16)<br>$PDA_{ES} \xleftrightarrow{\hspace{1cm}} CFG$<br>(Lecture 11/12)<br>$\vdots$<br><b>Chomsky Normal Form</b><br>(Lecture 18) | (Lecture 11) $CFL$<br>$\vdots$<br><b>Closure Properties</b><br>(Lecture 19)                         |
| <b>(Part 1)<br/>Finite<br/>Automata</b>   | (Lecture 4) $NFA \xleftrightarrow{\hspace{1cm}} DFA$<br>(Lecture 3) $DFA \xleftrightarrow{\hspace{1cm}} \epsilon\text{-NFA}$<br>(Lecture 5) $\epsilon\text{-NFA} \xleftrightarrow{\hspace{1cm}} RE$<br>(Lecture 7) $RE \xleftrightarrow{\hspace{1cm}} RE$<br>(Lecture 6)<br><b>Equivalence &amp; Minimization</b><br>(Lecture 10) |  | (Lecture 13) <b>Parse Trees &amp; Ambiguity</b><br>$\vdots$<br><b>Pumping Lemma</b><br>(Lecture 20) |
| <b>(Part 0)<br/>Basic<br/>Concepts</b>    | (Lecture 1)<br><b>Mathematical Preliminaries</b>  | (Lecture 2)<br><b>Scala</b>  |   |

- The final exam will be given in class.
- **Date:** 13:30-14:45 (1 hour 15 minutes), June 19 (Wed.).
- **Location:** 604, Woojung Hall of Informatics (우정정보관 604호)
- **Coverage:** Lectures 14 – 26
- **Format:** 7–9 questions with closed book and closed notes
  - Filling blanks in some tables, sentences, or expressions.
  - Construction of automata or grammars for given languages.
  - Proofs of given statements related to automata or grammars.
  - Yes/No questions about concepts in the theory of computation.
  - etc.
- Note that there is **no class** on **June 17 (Mon.)**.
- Please refer to the **previous exams** in the course website:

<https://plrg.korea.ac.kr/courses/cose215/>

- I hope you enjoyed the class!

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