

Lecture 27 – Course Review

COSE215: Theory of Computation

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2024 Spring

- What is the *mathematical model* of computers?

- What is the *mathematical model* of computers?

Turing Machine!

Let's learn **Turing Machine**

- What is the *mathematical model* of computers?

Turing Machine!

Let's learn **Turing Machine**

- Is it possible to solve *every problem* using computers?

- What is the *mathematical model* of computers?

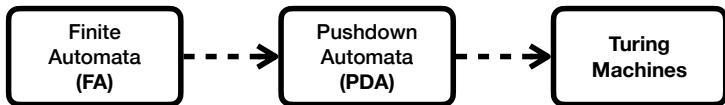
Turing Machine!

Let's learn **Turing Machine**

- Is it possible to solve *every problem* using computers?

No!

Let's learn **Undecidability** and **Intractability**



- **Finite Automata (FA)**
 - Regular Expressions and Languages
 - Applications: text search, etc.
- **Pushdown Automata (PDA)**
 - Context-Free Grammars (CFGs) and Languages (CFLs)
 - Applications: programming languages, natural language processing, etc.
- **Turing Machines (TMs)**
 - Extensions of Turing Machines
 - Undecidability and Intractability

	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata			
(Part 1) Finite Automata			
(Part 0) Basic Concepts			

	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata			
(Part 1) Finite Automata			
(Part 0) Basic Concepts	(Lecture 1) Mathematical Preliminaries		

	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata			
(Part 1) Finite Automata			
(Part 0) Basic Concepts	(Lecture 1) Mathematical Preliminaries	(Lecture 2) Scala	

	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata			
(Part 1) Finite Automata	(Lecture 3) DFA		(Lecture 3) RL
(Part 0) Basic Concepts	(Lecture 1) Mathematical Preliminaries	(Lecture 2) Scala	

	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata			
(Part 1) Finite Automata	(Lecture 4) NFA \longleftrightarrow (Lecture 3) DFA		(Lecture 3) RL
(Part 0) Basic Concepts	(Lecture 1) Mathematical Preliminaries	(Lecture 2) Scala	

	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata			
(Part 1) Finite Automata	(Lecture 4) NFA \longleftrightarrow (Lecture 3) DFA \longleftrightarrow (Lecture 5) ϵ -NFA		(Lecture 3) RL
(Part 0) Basic Concepts	(Lecture 1) Mathematical Preliminaries	(Lecture 2) Scala	

	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata			
(Part 1) Finite Automata	(Lecture 4) (Lecture 3) (Lecture 5) NFA \longleftrightarrow DFA \longleftrightarrow ϵ -NFA	(Lecture 6) RE	(Lecture 3) RL
(Part 0) Basic Concepts	(Lecture 1) Mathematical Preliminaries	(Lecture 2) Scala	

	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata			
(Part 1) Finite Automata	(Lecture 4) NFA	(Lecture 3) DFA	(Lecture 5) ϵ -NFA
	\longleftrightarrow	\longleftrightarrow	\longleftrightarrow
			(Lecture 7) RE
			(Lecture 6) RL
(Part 0) Basic Concepts	(Lecture 1) Mathematical Preliminaries		(Lecture 2) Scala

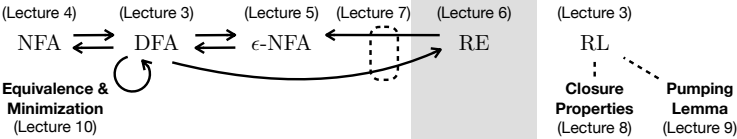
	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata			
(Part 1) Finite Automata	(Lecture 4) NFA	(Lecture 3) DFA	(Lecture 3) RL ⋮ Closure Properties (Lecture 8)
(Part 0) Basic Concepts	(Lecture 5) ε-NFA	(Lecture 6) RE	

Diagram illustrating the relationships between Automata, Grammars, and Languages across different parts of the course.

In the (Part 1) Finite Automata row, the following relationships are shown:

- NFA (Lecture 4) and DFA (Lecture 3) are connected by a double-headed arrow.
- DFA (Lecture 3) and ε-NFA (Lecture 5) are connected by a double-headed arrow.
- ε-NFA (Lecture 5) and RE (Lecture 6) are connected by a double-headed arrow.
- RE (Lecture 6) and RL (Lecture 3) are connected by a double-headed arrow.
- A dashed box encloses the ε-NFA and RE nodes.
- A curved arrow points from the ε-NFA node to the RE node.

	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata			
(Part 1) Finite Automata	(Lecture 4) NFA	(Lecture 3) DFA	(Lecture 5) ϵ -NFA
	↔	↔	↔
			(Lecture 7) RE
			(Lecture 6) ↖ (dashed arrow) ↗ (solid arrow)
			(Lecture 3) RL
			 Closure Properties (Lecture 8)
			 Pumping Lemma (Lecture 9)
(Part 0) Basic Concepts	(Lecture 1) Mathematical Preliminaries	(Lecture 2) Scala	

	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata			
(Part 1) Finite Automata	<p>(Lecture 4) NFA \rightleftarrows DFA \rightleftarrows ϵ-NFA (Lecture 3)</p> <p>Equivalence & Minimization (Lecture 10)</p> 	<p>(Lecture 6) RE</p> <p>(Lecture 3) RL</p> <p>Closure Properties (Lecture 8)</p> <p>Pumping Lemma (Lecture 9)</p>	
(Part 0) Basic Concepts	(Lecture 1) Mathematical Preliminaries	(Lecture 2) Scala	

	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata		(Lecture 11/12) CFG	(Lecture 11) CFL
(Part 1) Finite Automata	(Lecture 4) NFA \rightleftarrows DFA \rightleftarrows ϵ -NFA Equivalence & Minimization (Lecture 10)	(Lecture 5) ϵ -NFA \leftarrow RE (Lecture 7)	(Lecture 3) RL Closure Properties (Lecture 8) Pumping Lemma (Lecture 9)
(Part 0) Basic Concepts	(Lecture 1) Mathematical Preliminaries	(Lecture 2) Scala	

	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata		(Lecture 11/12) CFG	(Lecture 11) CFL ... (Lecture 13) Parse Trees & Ambiguity
(Part 1) Finite Automata	(Lecture 4) NFA \rightleftarrows (Lecture 3) DFA \rightleftarrows (Lecture 5) ϵ -NFA \leftarrow (Lecture 7) RE \rightarrow (Lecture 6) RE Equivalence & Minimization (Lecture 10)		(Lecture 3) RL Closure Properties (Lecture 8) ... Pumping Lemma (Lecture 9)
(Part 0) Basic Concepts	(Lecture 1) Mathematical Preliminaries	(Lecture 2) Scala	

	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata	(Lecture 14/15) PDA _{FS} PDA _{ES}	(Lecture 11/12) CFG	(Lecture 11) CFL ... (Lecture 13) Parse Trees & Ambiguity
(Part 1) Finite Automata	(Lecture 4) NFA ↔ (Lecture 3) DFA ↔ (Lecture 5) ϵ -NFA ↔ (Lecture 7) RE Equivalence & Minimization (Lecture 10)	(Lecture 6) RE	(Lecture 3) RL Closure Properties (Lecture 8) Pumping Lemma (Lecture 9)
(Part 0) Basic Concepts	(Lecture 1) Mathematical Preliminaries	(Lecture 2) Scala	

	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata	(Lecture 14/15) $PDA_{FS} \rightleftarrows PDA_{ES}$	(Lecture 16) \rightleftarrows	(Lecture 11/12) CFG (Lecture 11) CFL ... (Lecture 13) Parse Trees & Ambiguity
(Part 1) Finite Automata	(Lecture 4) NFA \rightleftarrows Equivalence & Minimization (Lecture 10)	(Lecture 3) DFA \rightleftarrows (Lecture 5) ϵ -NFA \rightleftarrows (Lecture 7) \rightleftarrows (Lecture 6) RE	(Lecture 3) RL Closure Properties (Lecture 8) ... Pumping Lemma (Lecture 9)
(Part 0) Basic Concepts	(Lecture 1) Mathematical Preliminaries	(Lecture 2) Scala	

	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata	<p>(Lecture 14/15)</p> $\text{PDA}_{\text{FS}} \begin{array}{c} \xleftrightarrow{\text{---}} \\ \xleftrightarrow{\text{---}} \\ \xleftrightarrow{\text{---}} \end{array} \text{PDA}_{\text{ES}}$ <p>\cup</p> $\text{DPDA}_{\text{FS}} \supset \text{DPDA}_{\text{ES}}$ <p>\cup (Lecture 17) $\not\subset$</p>	<p>(Lecture 16)</p> $\text{PDA}_{\text{ES}} \begin{array}{c} \xleftrightarrow{\text{---}} \\ \xleftrightarrow{\text{---}} \end{array} \text{CFG}$	<p>(Lecture 11/12)</p> <p>(Lecture 11) CFL ... (Lecture 13) Parse Trees & Ambiguity</p>
(Part 1) Finite Automata	<p>(Lecture 4)</p> $\text{NFA} \begin{array}{c} \xleftrightarrow{\text{---}} \\ \xleftrightarrow{\text{---}} \end{array} \text{DFA}$ <p>Equivalence & Minimization (Lecture 10)</p>	<p>(Lecture 3)</p> $\text{DFA} \begin{array}{c} \xleftrightarrow{\text{---}} \\ \xleftrightarrow{\text{---}} \end{array} \epsilon\text{-NFA}$ <p>(Lecture 5)</p> $\epsilon\text{-NFA} \begin{array}{c} \xleftrightarrow{\text{---}} \\ \xleftrightarrow{\text{---}} \end{array} \text{RE}$ <p>(Lecture 7)</p> <p>(Lecture 6)</p>	<p>(Lecture 3)</p> RL <p>Closure Properties (Lecture 8)</p> <p>Pumping Lemma (Lecture 9)</p>
(Part 0) Basic Concepts	<p>(Lecture 1)</p> <p>Mathematical Preliminaries</p>	<p>(Lecture 2)</p> <p>Scala</p>	

	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata	<p>(Lecture 14/15)</p> $\text{PDA}_{\text{FS}} \begin{array}{c} \xleftrightarrow{\text{---}} \\ \xleftrightarrow{\text{---}} \\ \xleftrightarrow{\text{---}} \end{array} \text{PDA}_{\text{ES}} \xleftrightarrow{\text{---}} \text{PDA}_{\text{ES}}$ <p>\cup</p> $\text{DPDA}_{\text{FS}} \supset \text{DPDA}_{\text{ES}}$ <p>\cup (Lecture 17) \cup</p>	<p>(Lecture 16)</p> $\text{PDA}_{\text{ES}} \xleftrightarrow{\text{---}} \text{CFG}$ <p>(Lecture 11/12)</p> \vdots <p>Chomsky Normal Form (Lecture 18)</p>	<p>(Lecture 11)</p> $\text{CFL} \dots$ <p>(Lecture 13)</p> <p>Parse Trees & Ambiguity</p>
(Part 1) Finite Automata	<p>(Lecture 4)</p> $\text{NFA} \xleftrightarrow{\text{---}} \text{DFA} \xleftrightarrow{\text{---}} \text{DFA}$ <p>(Lecture 3)</p> <p>Equivalence & Minimization (Lecture 10)</p>	<p>(Lecture 5)</p> $\text{DFA} \xleftrightarrow{\text{---}} \epsilon\text{-NFA} \xleftrightarrow{\text{---}} \epsilon\text{-NFA}$ <p>(Lecture 7)</p> <p>(Lecture 6)</p> $\epsilon\text{-NFA} \xleftrightarrow{\text{---}} \text{RE}$	<p>(Lecture 3)</p> RL <p>\vdots</p> <p>Closure Properties (Lecture 8)</p> <p>Pumping Lemma (Lecture 9)</p>
(Part 0) Basic Concepts	<p>(Lecture 1)</p> <p>Mathematical Preliminaries</p>	<p>(Lecture 2)</p> <p>Scala</p>	

	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata	<p>(Lecture 14/15)</p> $\text{PDA}_{\text{FS}} \begin{array}{c} \xleftrightarrow{\text{---}} \\ \xleftrightarrow{\text{---}} \\ \xleftrightarrow{\text{---}} \end{array} \text{PDA}_{\text{ES}} \xleftrightarrow{\text{---}} \text{PDA}_{\text{ES}}$ <p>\cup</p> $\text{DPDA}_{\text{FS}} \supset \text{DPDA}_{\text{ES}}$ <p>\cup (Lecture 17) $\not\subseteq$</p>	<p>(Lecture 16)</p> $\text{PDA}_{\text{ES}} \xleftrightarrow{\text{---}} \text{CFG}$ <p>(Lecture 11/12)</p> \vdots <p>Chomsky Normal Form (Lecture 18)</p>	<p>(Lecture 11)</p> $\text{CFL} \dots$ <p>(Lecture 13)</p> <p>Parse Trees & Ambiguity</p> <p>\vdots</p> <p>Closure Properties (Lecture 19)</p>
(Part 1) Finite Automata	<p>(Lecture 4)</p> $\text{NFA} \xleftrightarrow{\text{---}} \text{DFA} \xleftrightarrow{\text{---}} \epsilon\text{-NFA}$ <p>(Lecture 3)</p> <p>(Lecture 5)</p> <p>(Lecture 7)</p> <p>(Lecture 6)</p> <p>Equivalence & Minimization (Lecture 10)</p>	<p>(Lecture 6)</p> $\epsilon\text{-NFA} \xleftrightarrow{\text{---}} \text{RE}$	<p>(Lecture 3)</p> RL <p>\vdots</p> <p>Closure Properties (Lecture 8)</p> <p>Pumping Lemma (Lecture 9)</p>
(Part 0) Basic Concepts	<p>(Lecture 1)</p> <p>Mathematical Preliminaries</p>	<p>(Lecture 2)</p> <p>Scala</p>	

	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata	<p>(Lecture 14/15)</p> $\text{PDA}_{\text{FS}} \begin{matrix} \xrightarrow{\text{---}} \\ \xleftarrow{\text{---}} \end{matrix} \text{PDA}_{\text{ES}}$ <p>\cup</p> $\text{DPDA}_{\text{FS}} \supset \text{DPDA}_{\text{ES}}$ <p>\cup (Lecture 17) $\not\subset$</p>	<p>(Lecture 16)</p> $\text{PDA}_{\text{ES}} \begin{matrix} \xrightarrow{\text{---}} \\ \xleftarrow{\text{---}} \end{matrix} \text{CFG}$ <p>(Lecture 11/12)</p> <p>CFG</p> <p>⋮</p> <p>Chomsky Normal Form (Lecture 18)</p>	<p>(Lecture 11)</p> <p>CFL</p> <p>⋮</p> <p>Closure Properties (Lecture 19)</p> <p>(Lecture 13)</p> <p>Parse Trees & Ambiguity</p> <p>⋮</p> <p>Pumping Lemma (Lecture 20)</p>
(Part 1) Finite Automata	<p>(Lecture 4)</p> <p>NFA</p> <p>↔</p> <p>(Lecture 3)</p> <p>DFA</p> <p>↔</p> <p>(Lecture 5)</p> <p>ϵ-NFA</p> <p>↔</p> <p>(Lecture 7)</p> <p>RE</p> <p>(Lecture 6)</p> <p>↔</p> <p>Equivalence & Minimization (Lecture 10)</p>		<p>(Lecture 3)</p> <p>RL</p> <p>⋮</p> <p>Closure Properties (Lecture 8)</p> <p>⋮</p> <p>Pumping Lemma (Lecture 9)</p>
(Part 0) Basic Concepts	<p>(Lecture 1)</p> <p>Mathematical Preliminaries</p>	<p>(Lecture 2)</p> <p>Scala</p>	

	Automata	Grammars	Languages
(Part 3) Turing Machines	(Lecture 21/22) TM		(Lecture 21) REL
(Part 2) Pushdown Automata	(Lecture 14/15) $PDA_{FS} \leftrightarrow PDA_{ES}$ \cup $DPDA_{FS} \supset DPDA_{ES}$ \cup (Lecture 17)	(Lecture 16) \leftrightarrow CFG ⋮ Chomsky Normal Form (Lecture 18)	(Lecture 11) CFL ⋮ Closure Properties (Lecture 19) (Lecture 13) Parse Trees & Ambiguity ⋮ Pumping Lemma (Lecture 20)
(Part 1) Finite Automata	(Lecture 4) NFA \leftrightarrow (Lecture 3) DFA \leftrightarrow (Lecture 5) ϵ -NFA Equivalence & Minimization (Lecture 10) (Lecture 7) \leftrightarrow (Lecture 6) RE		(Lecture 3) RL ⋮ Closure Properties (Lecture 8) (Lecture 9) Pumping Lemma
(Part 0) Basic Concepts	(Lecture 1) Mathematical Preliminaries	(Lecture 2) Scala	

	Automata	Grammars	Languages
(Part 3) Turing Machines	(Lecture 23) ETM \longleftrightarrow (Lecture 21/22) TM		(Lecture 21) REL
(Part 2) Pushdown Automata	(Lecture 14/15) PDA _{FS} \longleftrightarrow PDA _{ES} \cup DPDA _{FS} \supset DPDA _{ES} \cup (Lecture 17) \curvearrowright	(Lecture 16) \longleftrightarrow (Lecture 11/12) CFG ⋮ Chomsky Normal Form (Lecture 18)	(Lecture 11) CFL ⋮ Closure Properties (Lecture 19) (Lecture 13) Parse Trees & Ambiguity ⋮ Pumping Lemma (Lecture 20)
(Part 1) Finite Automata	(Lecture 4) NFA \longleftrightarrow (Lecture 3) DFA \longleftrightarrow (Lecture 5) ϵ -NFA \longleftrightarrow (Lecture 7) RE Equivalence & Minimization (Lecture 10)	(Lecture 6)	(Lecture 3) RL ⋮ Closure Properties (Lecture 8) (Lecture 9) Pumping Lemma
(Part 0) Basic Concepts	(Lecture 1) Mathematical Preliminaries	(Lecture 2) Scala	

	Automata	Grammars	Languages
(Part 3) Turing Machines	(Lecture 23) $\text{ETM} \rightleftharpoons \text{TM}$ (Lecture 21/22)	(Lecture 24) $\text{TM} \rightleftharpoons \text{LC}$	(Lecture 21) REL
(Part 2) Pushdown Automata	(Lecture 14/15) $\text{PDA}_{\text{FS}} \rightleftharpoons \text{PDA}_{\text{ES}}$ \cup $\text{DPDA}_{\text{FS}} \supset \text{DPDA}_{\text{ES}}$ \cup (Lecture 17) \curvearrowright	(Lecture 16) $\text{PDA}_{\text{ES}} \rightleftharpoons \text{CFG}$ (Lecture 11/12) \vdots Chomsky Normal Form (Lecture 18)	(Lecture 11) CFL (Lecture 13) Parse Trees & Ambiguity \vdots Closure Properties (Lecture 19) Pumping Lemma (Lecture 20)
(Part 1) Finite Automata	(Lecture 4) $\text{NFA} \rightleftharpoons \text{DFA}$ (Lecture 3) (Lecture 5) $\text{DFA} \rightleftharpoons \epsilon\text{-NFA}$ (Lecture 7) (Lecture 6) $\epsilon\text{-NFA} \rightleftharpoons \text{RE}$ (Lecture 10) Equivalence & Minimization		(Lecture 3) RL \vdots Closure Properties (Lecture 8) Pumping Lemma (Lecture 9)
(Part 0) Basic Concepts	(Lecture 1) Mathematical Preliminaries	(Lecture 2) Scala	

	Automata	Grammars	Languages
(Part 3) Turing Machines	(Lecture 23) $ETM \rightleftharpoons TM$	(Lecture 21/22) $TM \rightleftharpoons LC$	(Lecture 24) REL \cup DL (Lecture 25)
(Part 2) Pushdown Automata	(Lecture 14/15) $PDA_{FS} \rightleftharpoons PDA_{ES}$ \cup $DPDA_{FS} \supset DPDA_{ES}$ \cup (Lecture 17)	(Lecture 16) $PDA_{ES} \rightleftharpoons CFG$ \vdots Chomsky Normal Form (Lecture 18)	(Lecture 11) CFL \vdots Closure Properties (Lecture 19) (Lecture 13) Parse Trees & Ambiguity \vdots Pumping Lemma (Lecture 20)
(Part 1) Finite Automata	(Lecture 4) NFA \rightleftharpoons (Lecture 3) DFA Equivalence & Minimization (Lecture 10)	(Lecture 5) ϵ -NFA \rightleftharpoons (Lecture 7) RE (Lecture 6)	(Lecture 3) RL \vdots Closure Properties (Lecture 8) Pumping Lemma (Lecture 9)
(Part 0) Basic Concepts	(Lecture 1) Mathematical Preliminaries	(Lecture 2) Scala	

	Automata	Grammars	Languages
(Part 3) Turing Machines	(Lecture 23) $ETM \xleftrightarrow{\quad} TM$ (Lecture 21/22)	(Lecture 24) $TM \xleftrightarrow{\quad} LC$	(Lecture 21) REL \cup $DL \supset NP \stackrel{?}{=} P$ (Lecture 25)
(Part 2) Pushdown Automata	(Lecture 14/15) $PDA_{FS} \xleftrightarrow{\quad} PDA_{ES}$ \cup $DPDA_{FS} \supset DPDA_{ES}$ (Lecture 17)	(Lecture 16) $PDA_{ES} \xleftrightarrow{\quad} CFG$ \vdots Chomsky Normal Form (Lecture 18)	(Lecture 11) CFL \vdots Closure Properties (Lecture 19)
(Part 1) Finite Automata	(Lecture 4) $NFA \xleftrightarrow{\quad} DFA$ (Lecture 3) $DFA \xleftrightarrow{\quad} \epsilon\text{-NFA}$ (Lecture 5) $\epsilon\text{-NFA} \xleftrightarrow{\quad} RE$ (Lecture 7) $RE \xleftrightarrow{\quad} DFA$ (Lecture 6) $RE \xleftrightarrow{\quad} PDA_{ES}$ Equivalence & Minimization (Lecture 10)		(Lecture 13) Parse Trees & Ambiguity \vdots Pumping Lemma (Lecture 20)
(Part 0) Basic Concepts	(Lecture 1) Mathematical Preliminaries	(Lecture 2) Scala	

- The final exam will be given in class.
- **Date:** 13:30-14:45 (1 hour 15 minutes), June 19 (Wed.).
- **Location:** 604, Woojung Hall of Informatics (우정정보관 604호)
- **Coverage:** Lectures 14 – 26
- **Format:** 7–9 questions with closed book and closed notes
 - Filling blanks in some tables, sentences, or expressions.
 - Construction of automata or grammars for given languages.
 - Proofs of given statements related to automata or grammars.
 - Yes/No questions about concepts in the theory of computation.
 - etc.
- Note that there is **no class** on **June 17 (Mon.)**.
- Please refer to the **previous exams** in the course website:

<https://plrg.korea.ac.kr/courses/cose215/>

- I hope you enjoyed the class!

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