Lecture 27 – Course Review

COSE215: Theory of Computation

Jihyeok Park

2025 Spring

Goal of This Course

PLRG

What is the *mathematical model* of computers?

Turing Machine!

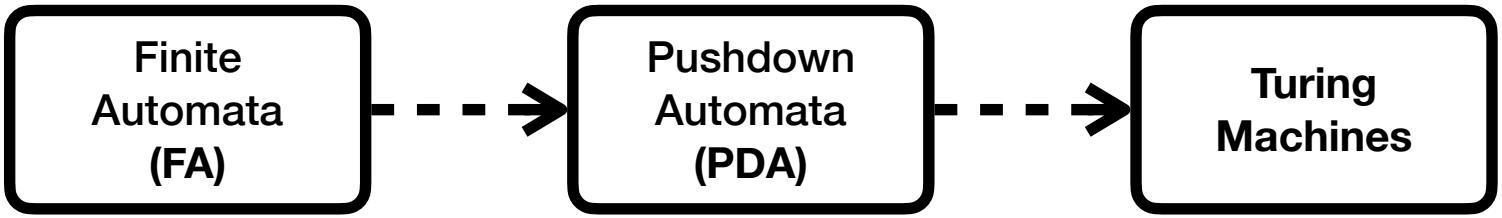
Let's learn **Turing Machine**

Is it possible to solve *every problem* using computers?

No!

Let's learn **Undecidability** and **Intractability**

Roadmap: Towards Turing Machine



Finite Automata (FA)

Regular Expressions and Languages
Applications: text search, etc.

Pushdown Automata (PDA)

Context-Free Grammars (CFGs) and Languages (CFLs)
Applications: programming languages, natural language processing, etc.

Turing Machines (TMs)

Extensions of Turing Machines
Undecidability and Intractability

Summary

	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata			
(Part 1) Finite Automata			
(Part 0) Basic Concepts			

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(Part 0) Basic Concepts	(Lecture 1) **Mathematical Preliminaries**	(Lecture 2) **Scala**	

Summary

PLRG

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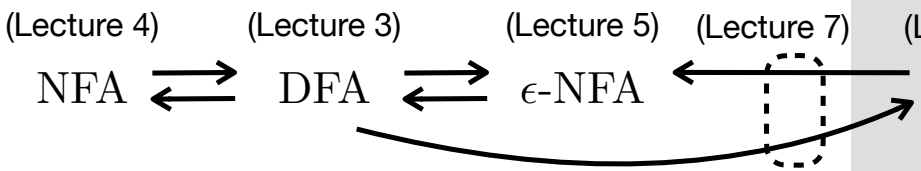
(Part 1) Finite Automata			(Lecture 3) RL **Closure Properties** (Lecture 8)
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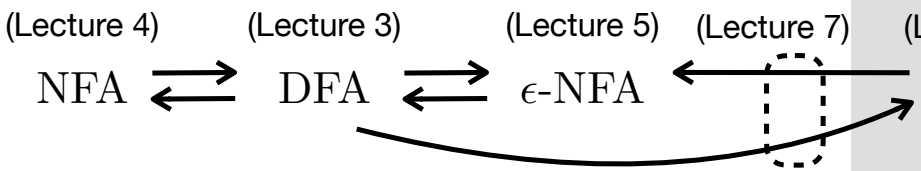
(Part 1) Finite Automata			(Lecture 3) RL **Closure Properties** (Lecture 8) **Pumping Lemma** (Lecture 9)
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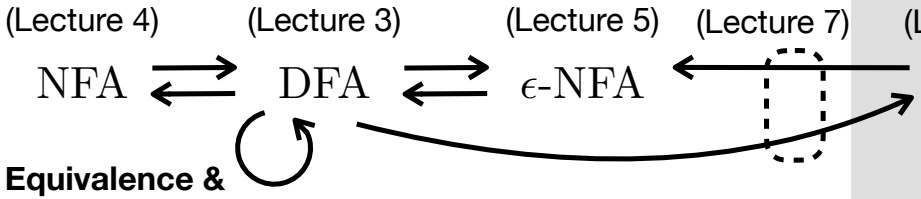
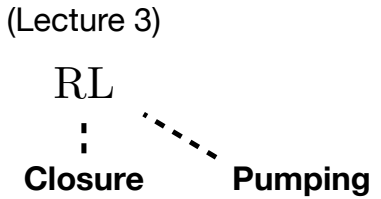
(Part 1) Finite Automata	 **Equivalence & Minimization** (Lecture 10)		 **Closure Properties** (Lecture 8) **Pumping Lemma** (Lecture 9)
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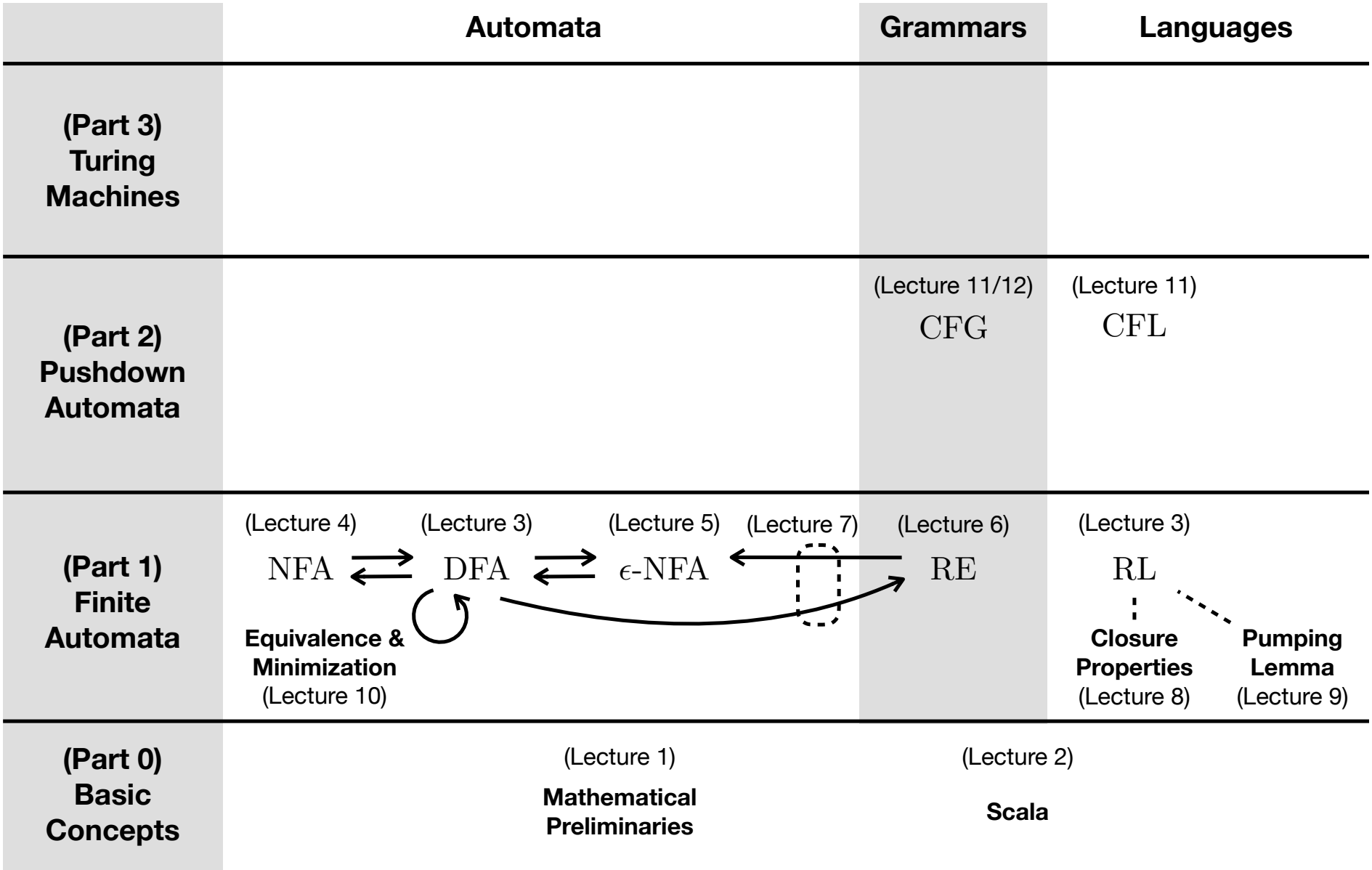
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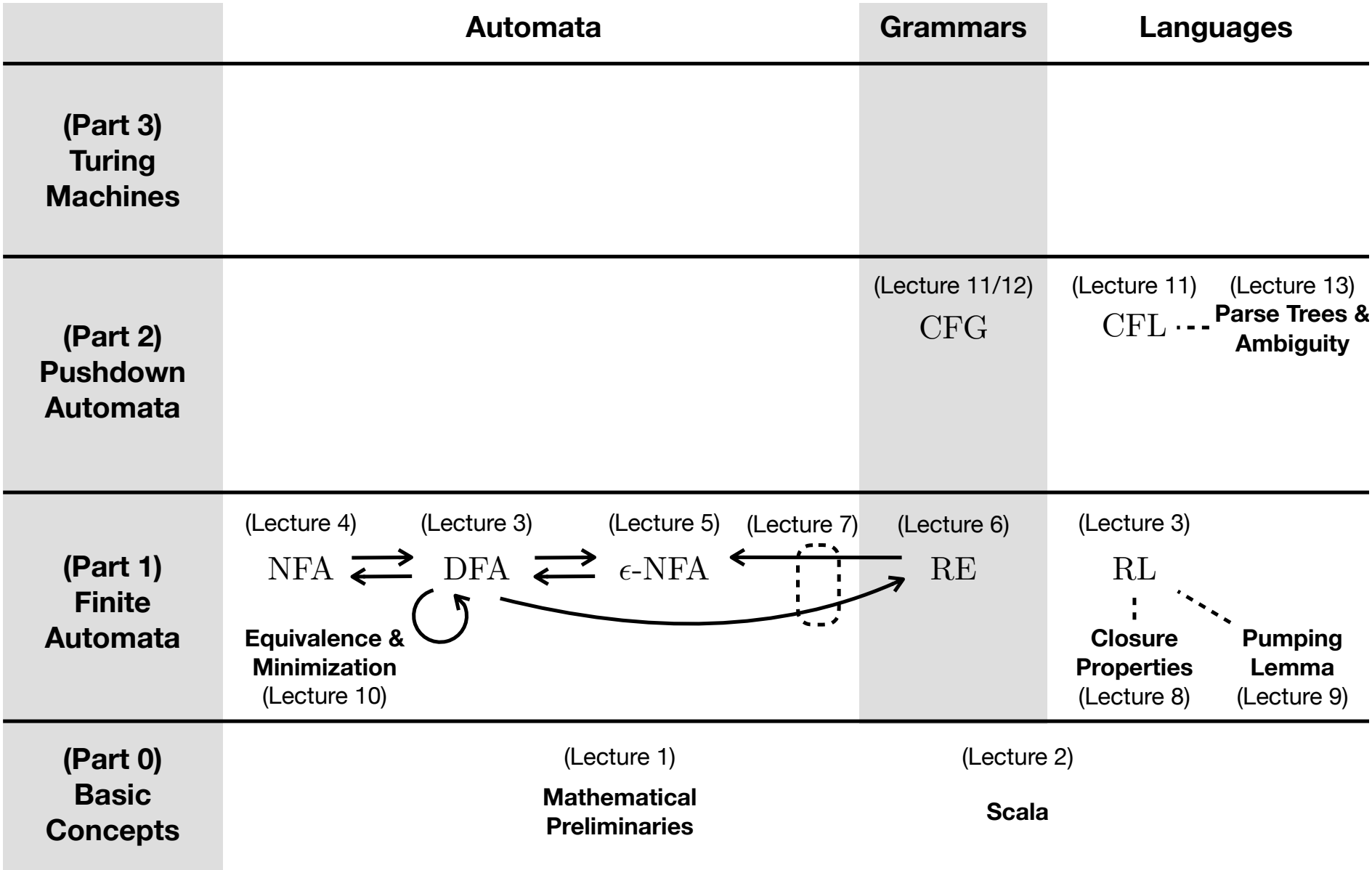
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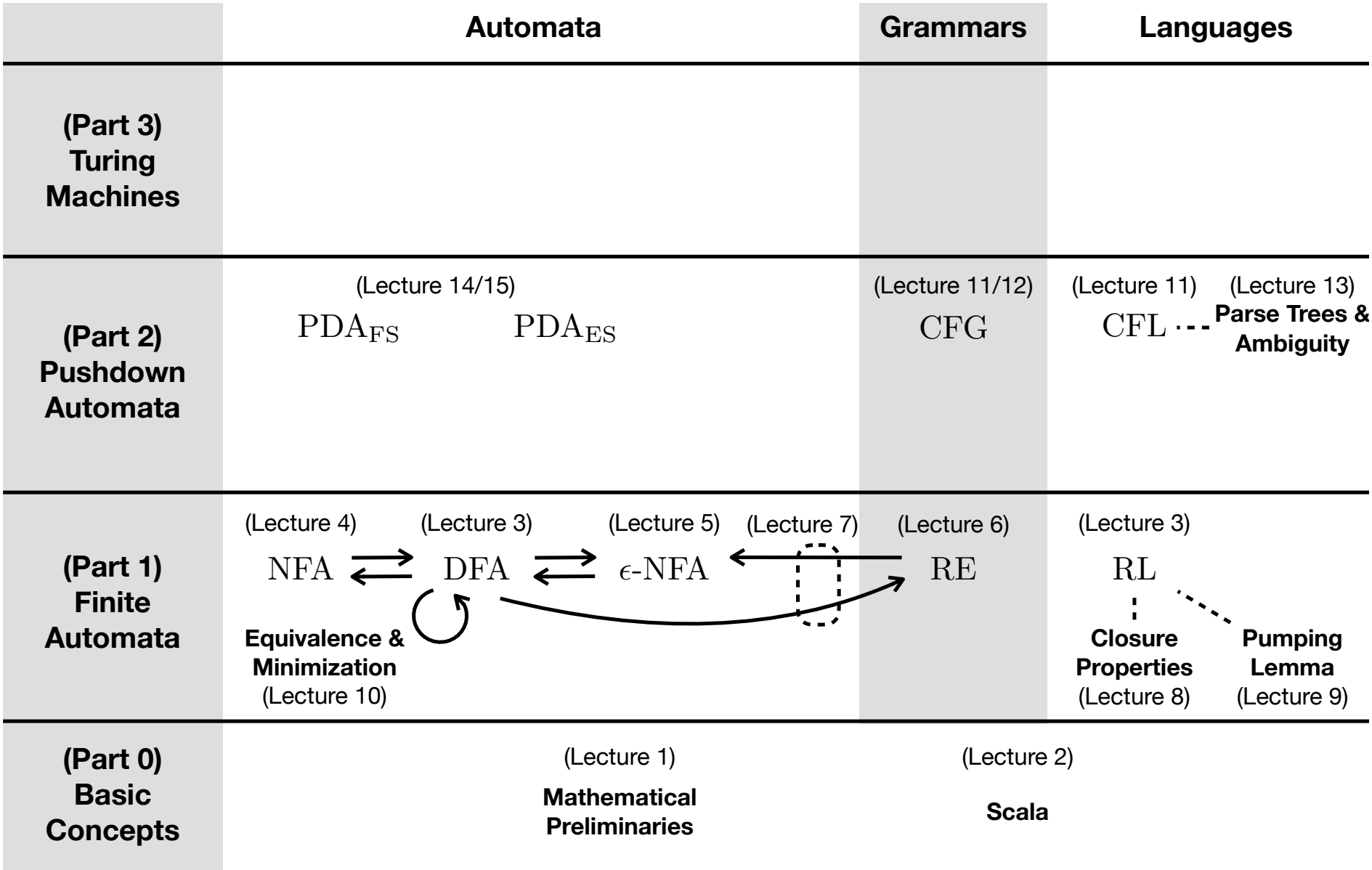
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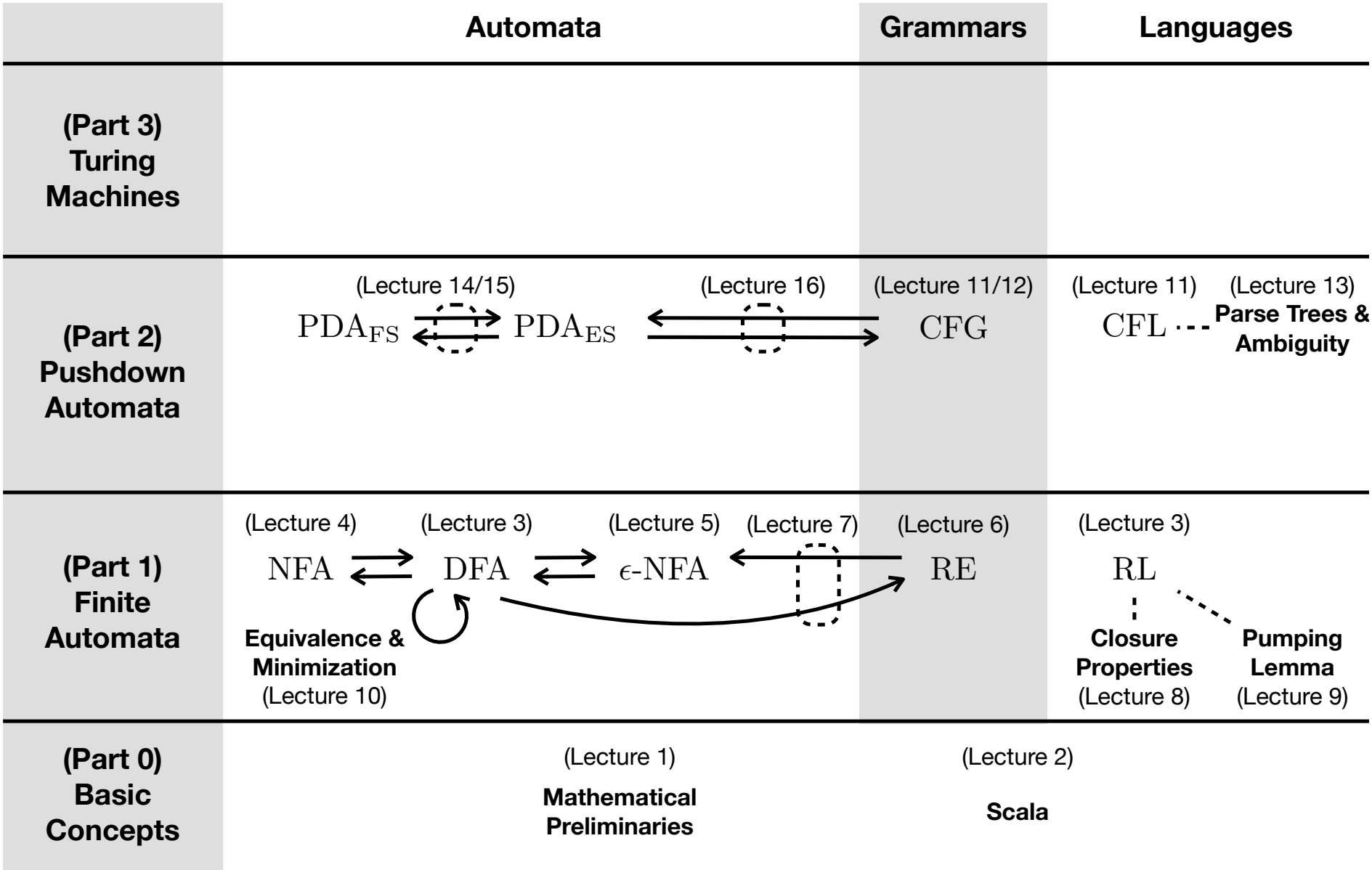
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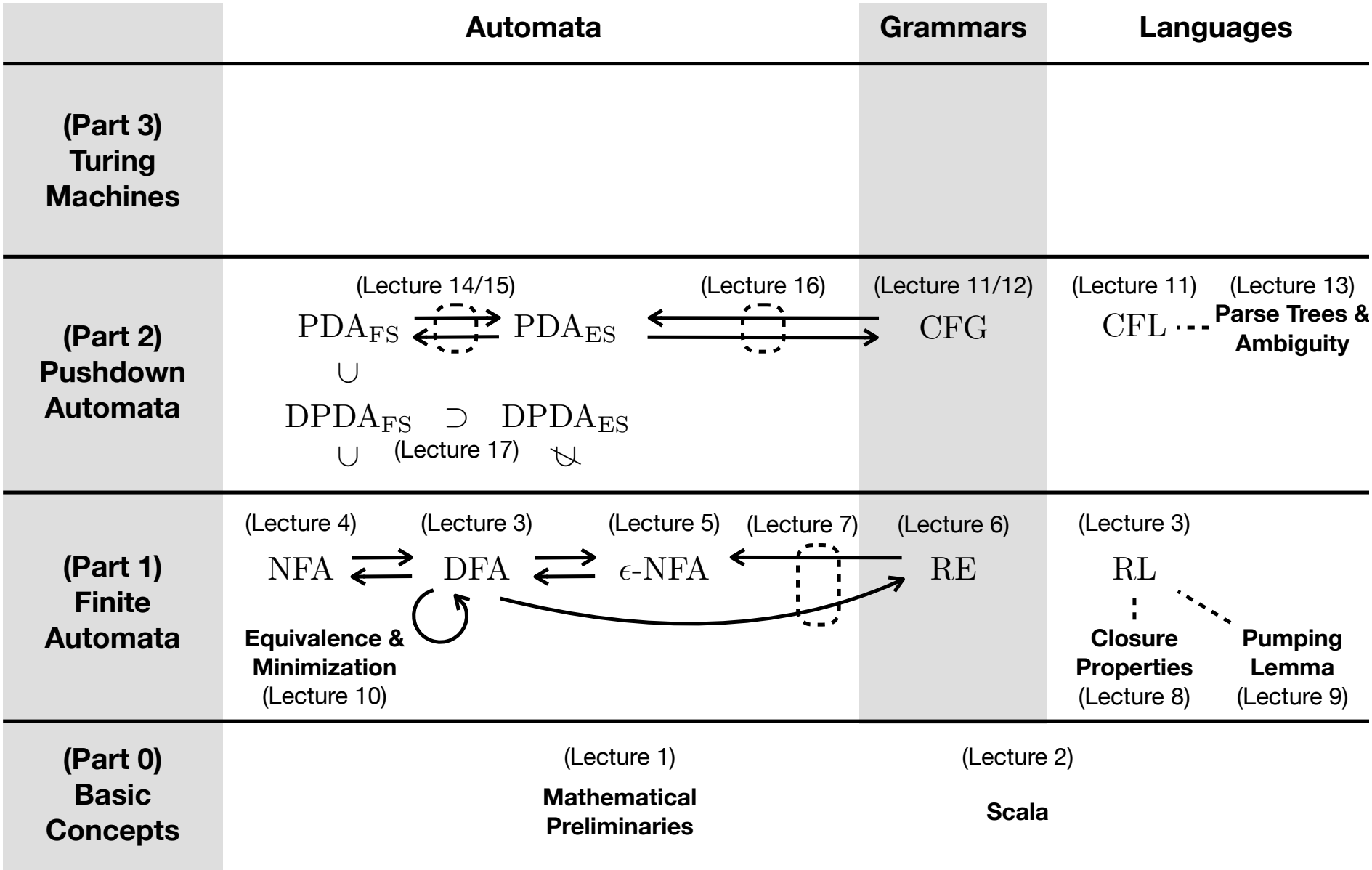
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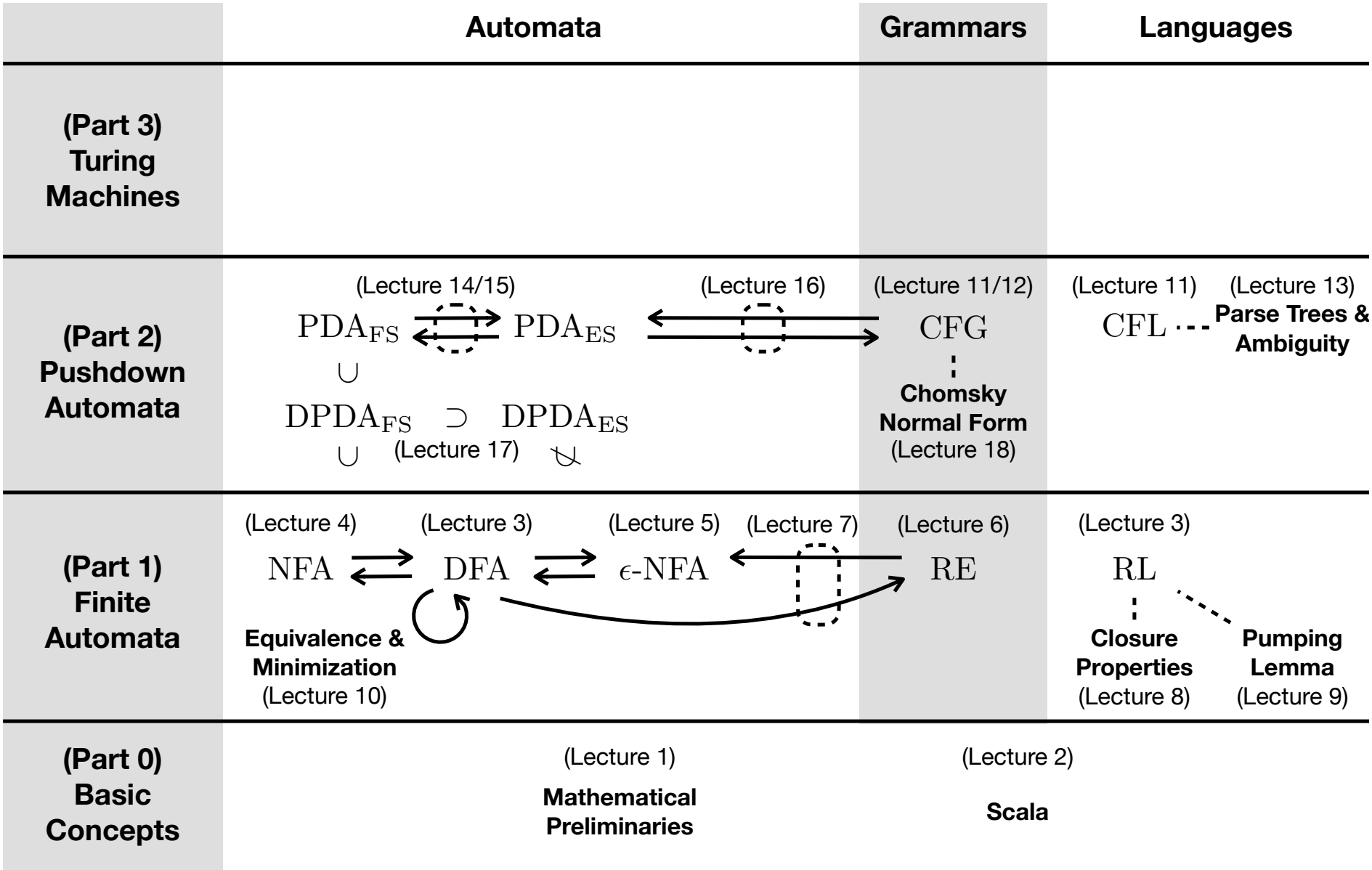
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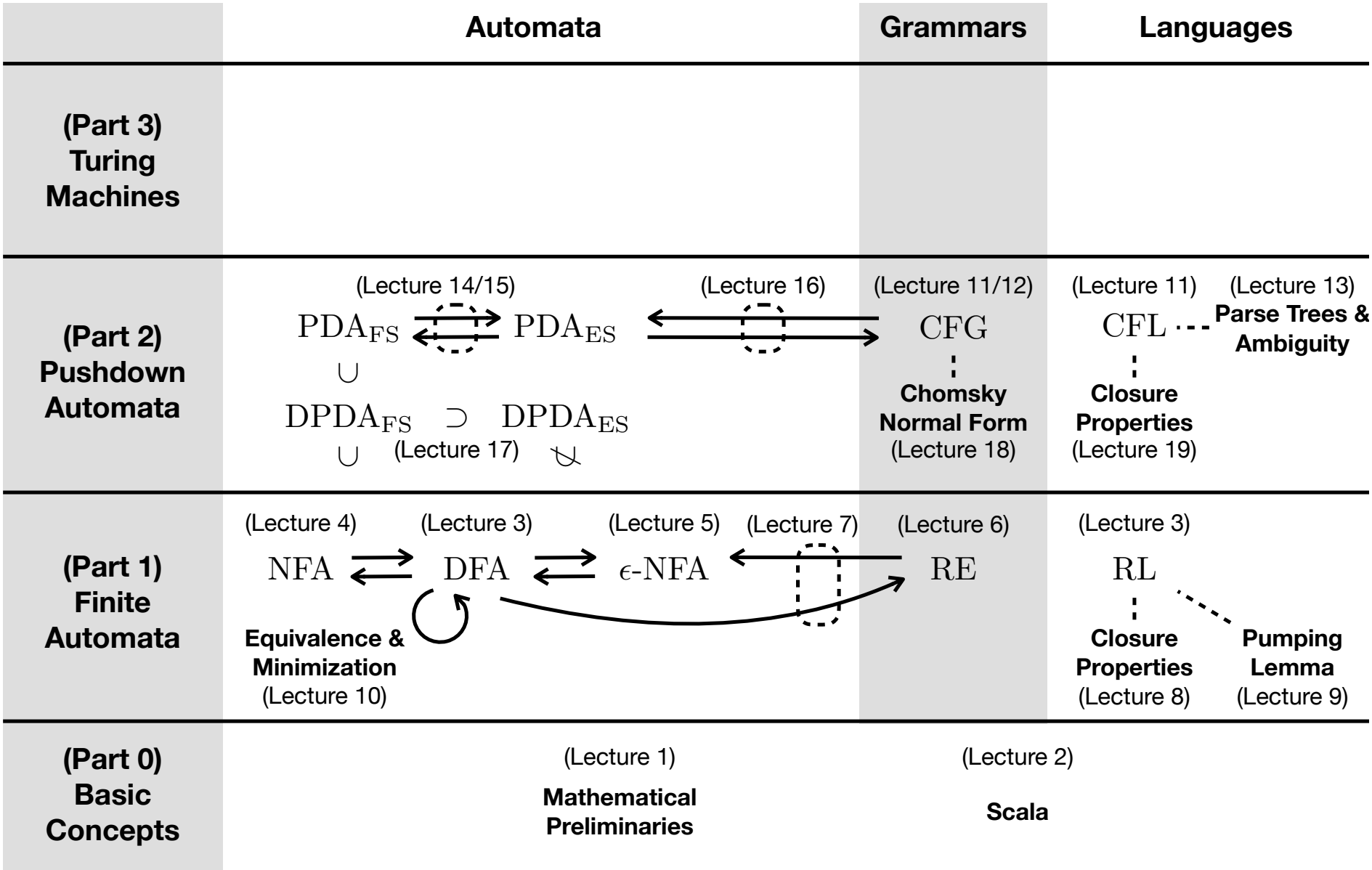
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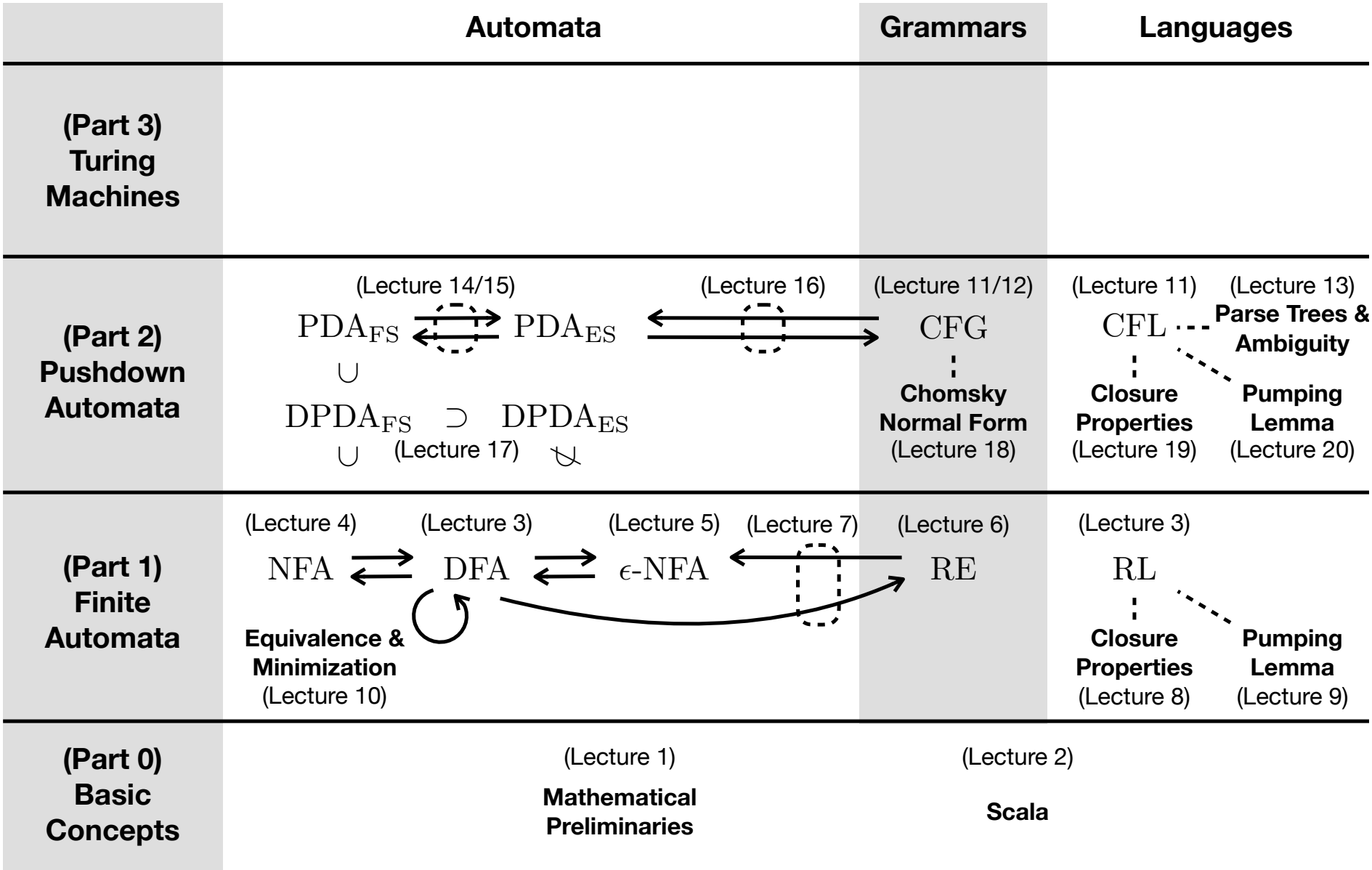
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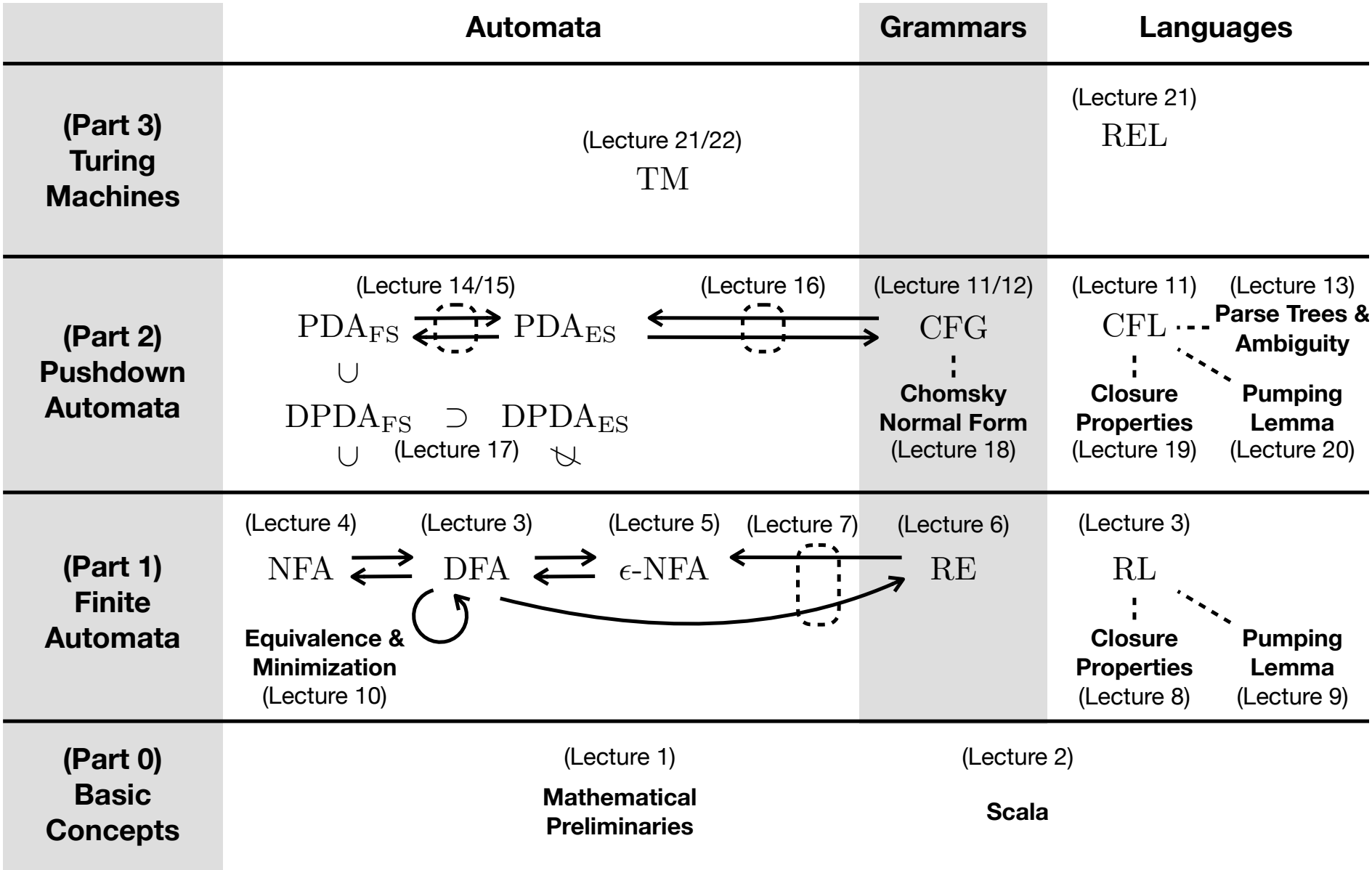
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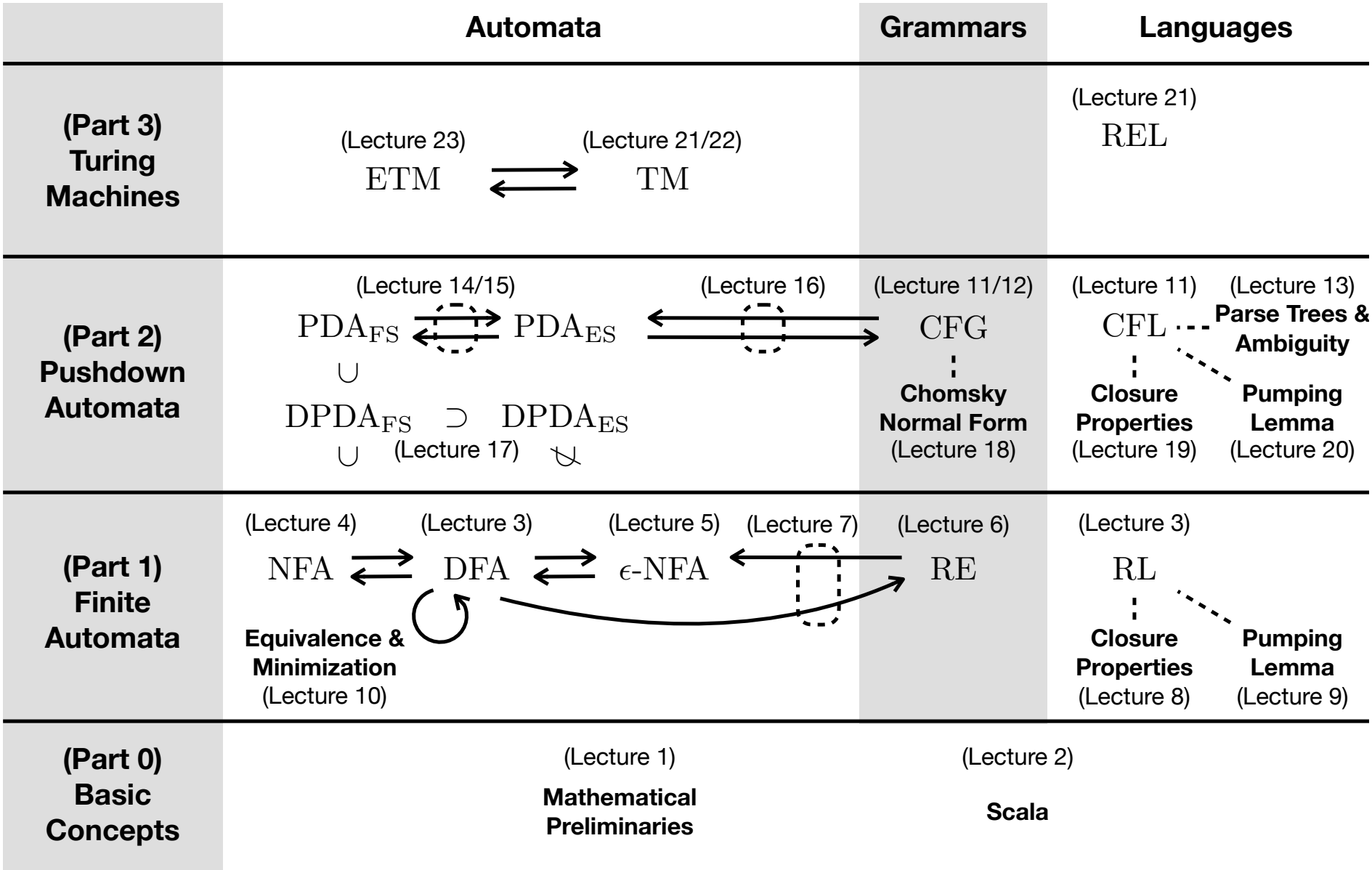
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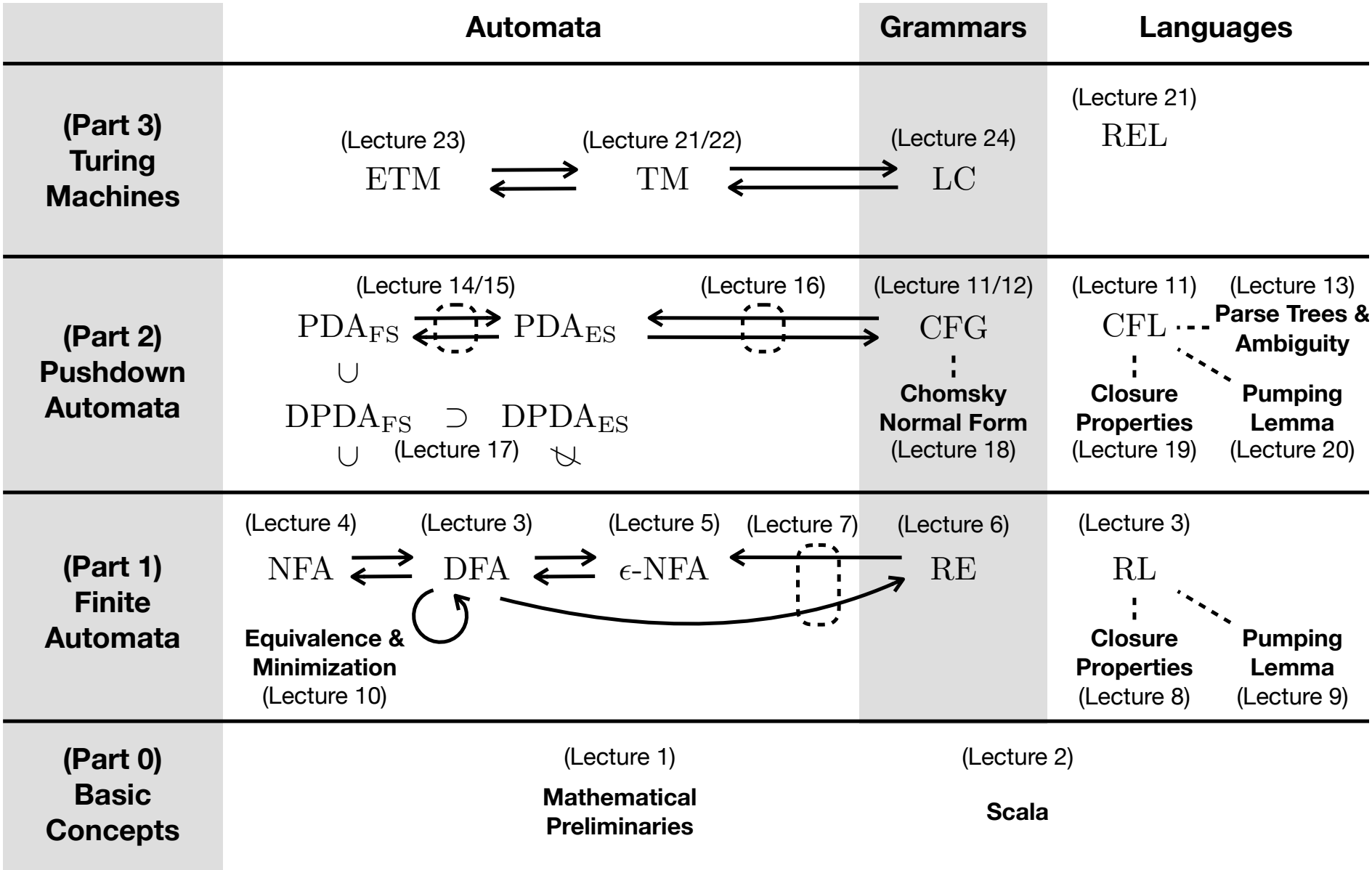
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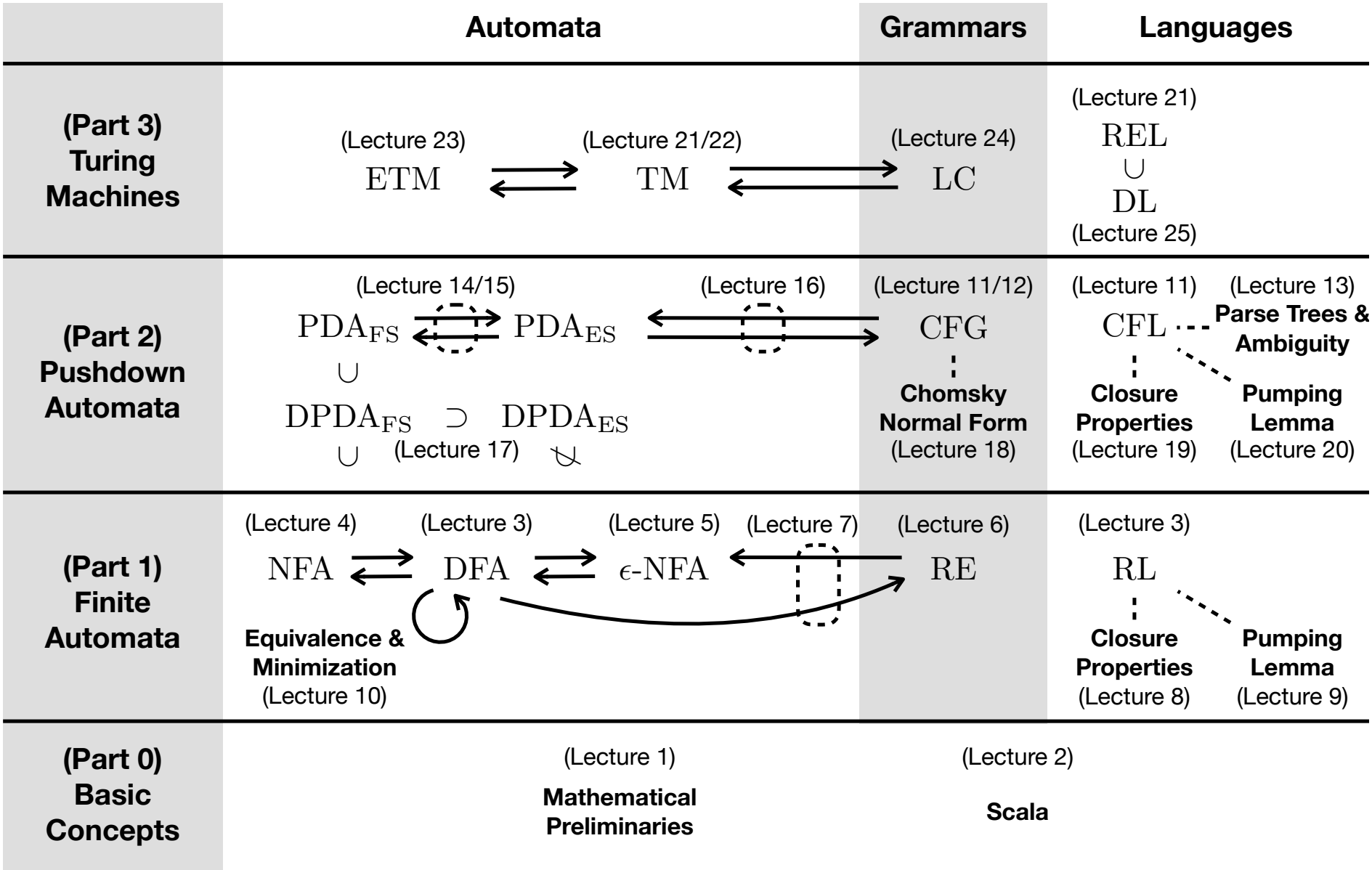
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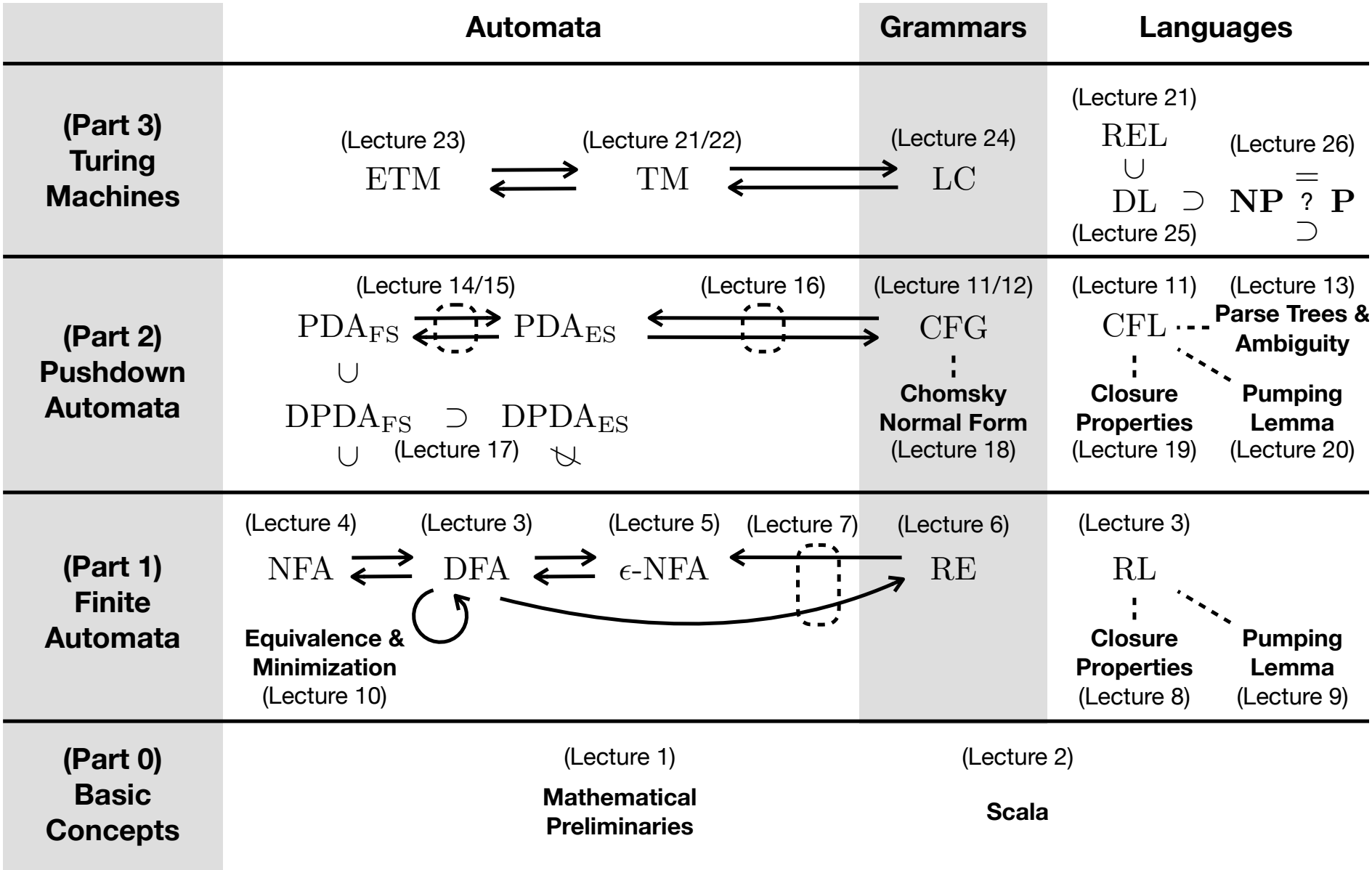
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Final Exam

The final exam will be given in class.

Date: 13:30-14:45 (1 hour 15 minutes), June 23 (Mon.).

Location: 301, Aegineung (애기능생활관 301호)

Coverage: Lectures 14 – 26

Format: 8 questions with closed book and closed notes

- Yes/No questions about concepts in the theory of computation.
- Construction of automata or grammars for given languages.
- Proofs of given statements related to automata or grammars.
- etc.

Note that there is **no class** on **June 18 (Wed.)**.

Please refer to the **previous exams** in the course website:

https://plrg.korea.ac.kr/courses/cose215/

I hope you enjoyed the class!

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https://plrg.korea.ac.kr