

Lecture 27 – Course Review

COSE215: Theory of Computation

Jihyeok Park



2025 Spring

What is the *mathematical model* of computers?

What is the *mathematical model* of computers?

Turing Machine!

Let's learn **Turing Machine**

What is the *mathematical model* of computers?

Turing Machine!

Let's learn **Turing Machine**

Is it possible to solve *every problem* using computers?

What is the *mathematical model* of computers?

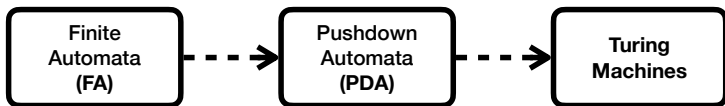
Turing Machine!

Let's learn **Turing Machine**

Is it possible to solve *every problem* using computers?

No!

Let's learn **Undecidability** and **Intractability**



Finite Automata (FA)

Regular Expressions and Languages

Applications: text search, etc.

Pushdown Automata (PDA)

Context-Free Grammars (CFGs) and Languages (CFLs)

Applications: programming languages, natural language processing, etc.

Turing Machines (TMs)

Extensions of Turing Machines

Undecidability and Intractability

	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata			
(Part 1) Finite Automata			
(Part 0) Basic Concepts			

	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata			
(Part 1) Finite Automata			
(Part 0) Basic Concepts	(Lecture 1) Mathematical Preliminaries		

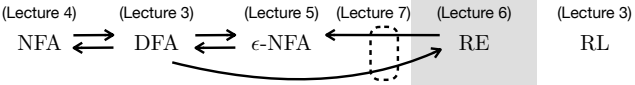
	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata			
(Part 1) Finite Automata			
(Part 0) Basic Concepts	(Lecture 1) Mathematical Preliminaries	(Lecture 2) Scala	

	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata			
(Part 1) Finite Automata	(Lecture 3) DFA		(Lecture 3) RL
(Part 0) Basic Concepts	(Lecture 1) Mathematical Preliminaries	(Lecture 2) Scala	

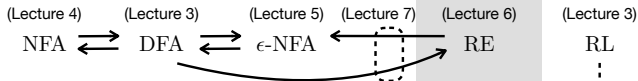
	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata			
(Part 1) Finite Automata	(Lecture 4) (Lecture 3) NFA $\longleftrightarrow$ DFA		(Lecture 3) RL
(Part 0) Basic Concepts	(Lecture 1) Mathematical Preliminaries	(Lecture 2) Scala	

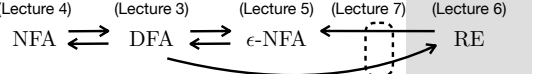
	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata			
(Part 1) Finite Automata	(Lecture 4) (Lecture 3) (Lecture 5) NFA $\longleftrightarrow$ DFA $\longleftrightarrow$ ϵ -NFA		(Lecture 3) RL
(Part 0) Basic Concepts	(Lecture 1) Mathematical Preliminaries	(Lecture 2) Scala	

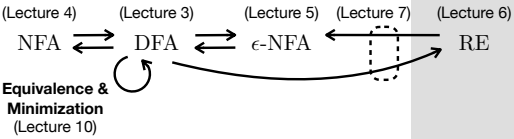
	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata			
(Part 1) Finite Automata	(Lecture 4) NFA $\longleftrightarrow$ (Lecture 3) DFA $\longleftrightarrow$ (Lecture 5) ϵ -NFA	(Lecture 6) RE	(Lecture 3) RL
(Part 0) Basic Concepts	(Lecture 1) Mathematical Preliminaries	(Lecture 2) Scala	

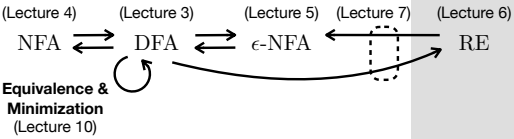
	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata			
(Part 1) Finite Automata	(Lecture 4) NFA ↔ (Lecture 3) DFA ↔ (Lecture 5) ϵ-NFA ↔ (Lecture 7) RE (Lecture 6) RL (Lecture 3) 		
(Part 0) Basic Concepts	(Lecture 1) Mathematical Preliminaries	(Lecture 2) Scala	

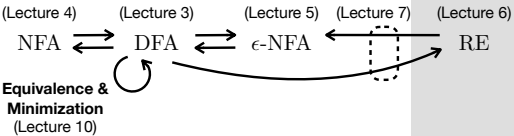
	Automata		Grammars	Languages
(Part 3) Turing Machines				
(Part 2) Pushdown Automata				
(Part 1) Finite Automata	(Lecture 4) NFA	(Lecture 3) DFA	(Lecture 5) ϵ -NFA	(Lecture 6) RE
			(Lecture 7)	(Lecture 3) RL : Closure Properties (Lecture 8)
(Part 0) Basic Concepts	(Lecture 1) Mathematical Preliminaries		(Lecture 2) Scala	



	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata			
(Part 1) Finite Automata	(Lecture 4) NFA $\longleftrightarrow$ (Lecture 3) DFA $\longleftrightarrow$ (Lecture 5) ϵ-NFA $\longleftrightarrow$ (Lecture 7) RE (Lecture 6) 		(Lecture 3) RL Closure Properties (Lecture 8) Pumping Lemma (Lecture 9)
(Part 0) Basic Concepts	(Lecture 1) Mathematical Preliminaries	(Lecture 2) Scala	

	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata			
(Part 1) Finite Automata	(Lecture 4) NFA $\longleftrightarrow$ (Lecture 3) DFA $\longleftrightarrow$ (Lecture 5) ϵ-NFA $\longleftrightarrow$ (Lecture 7) RE (Lecture 6) Equivalence & Minimization (Lecture 10) 		(Lecture 3) RL Closure Properties (Lecture 8) Pumping Lemma (Lecture 9)
(Part 0) Basic Concepts	(Lecture 1) Mathematical Preliminaries	(Lecture 2) Scala	

	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata		(Lecture 11/12) CFG	(Lecture 11) CFL
(Part 1) Finite Automata	(Lecture 4) NFA $\longleftrightarrow$ (Lecture 3) DFA $\longleftrightarrow$ (Lecture 5) ϵ-NFA $\longleftrightarrow$ (Lecture 7) RE (Lecture 6) Equivalence & Minimization (Lecture 10) 		(Lecture 3) RL Closure Properties (Lecture 8) Pumping Lemma (Lecture 9)
(Part 0) Basic Concepts	(Lecture 1) Mathematical Preliminaries	(Lecture 2) Scala	

	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata		(Lecture 11/12) CFG	(Lecture 11) CFL ... (Lecture 13) Parse Trees & Ambiguity
(Part 1) Finite Automata	(Lecture 4) NFA $\longleftrightarrow$ (Lecture 3) DFA $\longleftrightarrow$ (Lecture 5) ϵ-NFA $\longleftrightarrow$ (Lecture 7) RE (Lecture 6) Equivalence & Minimization (Lecture 10) 		(Lecture 3) RL ┆ Closure Properties (Lecture 8) ... Pumping Lemma (Lecture 9)
(Part 0) Basic Concepts	(Lecture 1) Mathematical Preliminaries	(Lecture 2) Scala	

	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata	(Lecture 14/15) PDA_{FS} PDA_{ES}	(Lecture 11/12) CFG	(Lecture 11) CFL ... (Lecture 13) Parse Trees & Ambiguity
(Part 1) Finite Automata	(Lecture 4) NFA $\longleftrightarrow$ (Lecture 3) DFA $\longleftrightarrow$ (Lecture 5) ϵ -NFA $\longleftrightarrow$ (Lecture 7) RE (Lecture 6) RE Equivalence & Minimization (Lecture 10)		(Lecture 3) RL Closure Properties (Lecture 8) Pumping Lemma (Lecture 9)
(Part 0) Basic Concepts	(Lecture 1) Mathematical Preliminaries	(Lecture 2) Scala	

	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata	(Lecture 14/15) $PDA_{FS} \longleftrightarrow PDA_{ES}$ (Lecture 16) $\longleftrightarrow$	(Lecture 11/12) CFG	(Lecture 11) CFL ... (Lecture 13) Parse Trees & Ambiguity
(Part 1) Finite Automata	(Lecture 4) NFA $\longleftrightarrow$ (Lecture 3) DFA $\longleftrightarrow$ (Lecture 5) ϵ-NFA $\longleftrightarrow$ (Lecture 7) RE (Lecture 6) Equivalence & Minimization (Lecture 10)		(Lecture 3) RL Closure Properties (Lecture 8) Pumping Lemma (Lecture 9)
(Part 0) Basic Concepts	(Lecture 1) Mathematical Preliminaries	(Lecture 2) Scala	

	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata	(Lecture 14/15) $\text{PDA}_{\text{FS}} \xleftrightarrow{\quad} \text{PDA}_{\text{ES}}$ $\cup$ $\text{DPDA}_{\text{FS}} \supset \text{DPDA}_{\text{ES}}$ $\cup$ (Lecture 17) $\nsubseteq$	(Lecture 16) $\text{PDA}_{\text{ES}} \xleftrightarrow{\quad} \text{CFG}$ (Lecture 11/12)	(Lecture 11) $\text{CFL} \dots$ (Lecture 13) Parse Trees & Ambiguity
(Part 1) Finite Automata	(Lecture 4) $\text{NFA} \xleftrightarrow{\quad} \text{DFA}$ (Lecture 3) $\text{DFA} \xleftrightarrow{\quad} \epsilon\text{-NFA}$ (Lecture 5) $\epsilon\text{-NFA} \xleftrightarrow{\quad} \text{RE}$ (Lecture 7) (Lecture 6) Equivalence & Minimization (Lecture 10)		(Lecture 3) RL Closure Properties (Lecture 8) Pumping Lemma (Lecture 9)
(Part 0) Basic Concepts	(Lecture 1) Mathematical Preliminaries	(Lecture 2) Scala	

	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata	(Lecture 14/15) $\text{PDA}_{\text{FS}} \xleftrightarrow{\text{dashed}} \text{PDA}_{\text{ES}} \xleftrightarrow{\text{dashed}} \text{CFG}$ $\cup$ $\text{DPDA}_{\text{FS}} \supset \text{DPDA}_{\text{ES}}$ $\cup$ (Lecture 17) $\mathcal{U}$	(Lecture 16) (Lecture 11/12) CFG ⋮ Chomsky Normal Form (Lecture 18)	(Lecture 11) CFL ... (Lecture 13) Parse Trees & Ambiguity
(Part 1) Finite Automata	(Lecture 4) NFA $\xleftrightarrow{\text{dashed}} \text{DFA} \xleftrightarrow{\text{dashed}} \epsilon\text{-NFA} \xleftrightarrow{\text{dashed}} \text{RE}$ (Lecture 3) (Lecture 5) (Lecture 7) (Lecture 6) Equivalence & Minimization (Lecture 10)		(Lecture 3) RL ⋮ Closure Properties (Lecture 8) Pumping Lemma (Lecture 9)
(Part 0) Basic Concepts	(Lecture 1) Mathematical Preliminaries	(Lecture 2) Scala	

	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata	(Lecture 14/15) $\text{PDA}_{\text{FS}} \xleftrightarrow{\quad} \text{PDA}_{\text{ES}}$ $\cup$ $\text{DPDA}_{\text{FS}} \supset \text{DPDA}_{\text{ES}}$ $\cup$ (Lecture 17) $\nsubseteq$	(Lecture 16) $\text{PDA}_{\text{FS}} \xleftrightarrow{\quad} \text{CFG}$ (Lecture 11/12) CFG ⋮ Chomsky Normal Form (Lecture 18)	(Lecture 11) CFL ... Parse Trees & Ambiguity ⋮ Closure Properties (Lecture 19)
(Part 1) Finite Automata	(Lecture 4) (Lecture 3) (Lecture 5) (Lecture 7) (Lecture 6) $\text{NFA} \xleftrightarrow{\quad} \text{DFA} \xleftrightarrow{\quad} \epsilon\text{-NFA} \xleftrightarrow{\quad} \text{RE}$ Equivalence & Minimization (Lecture 10)		(Lecture 3) RL ⋮ Closure Properties (Lecture 8) Pumping Lemma (Lecture 9)
(Part 0) Basic Concepts	(Lecture 1) Mathematical Preliminaries	(Lecture 2) Scala	

	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata	(Lecture 14/15) $\text{PDA}_{\text{FS}} \xleftrightarrow{\quad} \text{PDA}_{\text{ES}}$ $\cup$ $\text{DPDA}_{\text{FS}} \supset \text{DPDA}_{\text{ES}}$ $\cup$ (Lecture 17) $\nsubseteq$	(Lecture 16) $\text{PDA}_{\text{FS}} \xleftrightarrow{\quad} \text{CFG}$ (Lecture 11/12) CFG ⋮ Chomsky Normal Form (Lecture 18)	(Lecture 11) CFL ⋮ Closure Properties (Lecture 19) (Lecture 13) Parse Trees & Ambiguity ⋮ Pumping Lemma (Lecture 20)
(Part 1) Finite Automata	(Lecture 4) NFA ⋮ Equivalence & Minimization (Lecture 10)	(Lecture 3) DFA ⋮ (Lecture 5) ϵ-NFA (Lecture 7) RE	(Lecture 6) RE ⋮ (Lecture 3) RL ⋮ Closure Properties (Lecture 8) ⋮ Pumping Lemma (Lecture 9)
(Part 0) Basic Concepts	(Lecture 1) Mathematical Preliminaries	(Lecture 2) Scala	

	Automata	Grammars	Languages
(Part 3) Turing Machines	(Lecture 21/22) TM		(Lecture 21) REL
(Part 2) Pushdown Automata	(Lecture 14/15) $PDA_{FS} \longleftrightarrow PDA_{ES}$ $\cup$ $DPDA_{FS} \supset DPDA_{ES}$ $\cup$ (Lecture 17) $\mathcal{U}$	(Lecture 16) $\longleftrightarrow$ (Lecture 11/12) CFG Chomsky Normal Form (Lecture 18)	(Lecture 11) CFL (Lecture 13) Parse Trees & Ambiguity Closure Properties (Lecture 19) Pumping Lemma (Lecture 20)
(Part 1) Finite Automata	(Lecture 4) NFA $\longleftrightarrow$ (Lecture 3) DFA $\longleftrightarrow$ (Lecture 5) ϵ-NFA $\longleftrightarrow$ (Lecture 7) RE Equivalence & Minimization (Lecture 10)	(Lecture 6) RE	(Lecture 3) RL Closure Properties (Lecture 8) Pumping Lemma (Lecture 9)
(Part 0) Basic Concepts	(Lecture 1) Mathematical Preliminaries	(Lecture 2) Scala	

	Automata	Grammars	Languages
(Part 3) Turing Machines	(Lecture 23) ETM $\longleftrightarrow$ (Lecture 21/22) TM		(Lecture 21) REL
(Part 2) Pushdown Automata	(Lecture 14/15) $PDA_{FS} \longleftrightarrow PDA_{ES}$ $\cup$ $DPDA_{FS} \supset DPDA_{ES}$ $\cup$ (Lecture 17) $\nsubseteq$	(Lecture 16) $\longleftrightarrow$ (Lecture 11/12) CFG $\vdots$ Chomsky Normal Form (Lecture 18)	(Lecture 11) CFL $\cdots$ (Lecture 13) Parse Trees & Ambiguity $\vdots$ Closure Properties (Lecture 19) $\cdots$ Pumping Lemma (Lecture 20)
(Part 1) Finite Automata	(Lecture 4) NFA $\longleftrightarrow$ (Lecture 3) DFA $\longleftrightarrow$ (Lecture 5) ϵ -NFA $\longleftrightarrow$ (Lecture 7) RE Equivalence & Minimization (Lecture 10)	(Lecture 6) RE	(Lecture 3) RL $\cdots$ Pumping Lemma (Lecture 9) $\vdots$ Closure Properties (Lecture 8)
(Part 0) Basic Concepts	(Lecture 1) Mathematical Preliminaries	(Lecture 2) Scala	

	Automata	Grammars	Languages
(Part 3) Turing Machines	(Lecture 23) ETM $\longleftrightarrow$ (Lecture 21/22) TM $\longleftrightarrow$ (Lecture 24) LC		(Lecture 21) REL
(Part 2) Pushdown Automata	(Lecture 14/15) $PDA_{FS} \longleftrightarrow PDA_{ES}$ $\cup$ $DPDA_{FS} \supset DPDA_{ES}$ $\cup$ (Lecture 17) $\nsubseteq$	(Lecture 16) $\longleftrightarrow$ (Lecture 11/12) CFG : Chomsky Normal Form (Lecture 18)	(Lecture 11) CFL : Closure Properties (Lecture 19) (Lecture 13) Parse Trees & Ambiguity : Pumping Lemma (Lecture 20)
(Part 1) Finite Automata	(Lecture 4) NFA $\longleftrightarrow$ (Lecture 3) DFA $\longleftrightarrow$ (Lecture 5) ϵ -NFA $\longleftrightarrow$ (Lecture 7) RE Equivalence & Minimization (Lecture 10)	(Lecture 6) RE	(Lecture 3) RL : Closure Properties (Lecture 8) (Lecture 9) Pumping Lemma
(Part 0) Basic Concepts	(Lecture 1) Mathematical Preliminaries	(Lecture 2) Scala	

	Automata	Grammars	Languages
(Part 3) Turing Machines	(Lecture 23) ETM $\longleftrightarrow$ (Lecture 21/22) TM $\longleftrightarrow$ (Lecture 24) LC		(Lecture 21) REL $\cup$ DL (Lecture 25)
(Part 2) Pushdown Automata	(Lecture 14/15) $PDA_{FS} \longleftrightarrow PDA_{ES}$ $\cup$ $DPDA_{FS} \supset DPDA_{ES}$ $\cup$ (Lecture 17) $\nsubseteq$	(Lecture 16) $\longleftrightarrow$ (Lecture 11/12) CFG $\vdots$ Chomsky Normal Form (Lecture 18)	(Lecture 11) CFL $\cdots$ (Lecture 13) Parse Trees & Ambiguity $\vdots$ Closure Properties (Lecture 19) $\cdots$ Pumping Lemma (Lecture 20)
(Part 1) Finite Automata	(Lecture 4) NFA $\longleftrightarrow$ (Lecture 3) DFA $\longleftrightarrow$ (Lecture 5) ϵ -NFA $\longleftrightarrow$ (Lecture 7) RE Equivalence & Minimization (Lecture 10)	(Lecture 6) RE	(Lecture 3) RL $\vdots$ Closure Properties (Lecture 8) $\cdots$ Pumping Lemma (Lecture 9)
(Part 0) Basic Concepts	(Lecture 1) Mathematical Preliminaries	(Lecture 2) Scala	

	Automata	Grammars	Languages
(Part 3) Turing Machines	(Lecture 23) ETM $\longleftrightarrow$ (Lecture 21/22) TM $\longleftrightarrow$ (Lecture 24) LC		(Lecture 21) REL $\cup$ DL $\supset$ NP $\stackrel{?}{=} P$ (Lecture 26) (Lecture 25)
(Part 2) Pushdown Automata	(Lecture 14/15) $PDA_{FS} \longleftrightarrow PDA_{ES}$ $\cup$ $DPDA_{FS} \supset DPDA_{ES}$ $\cup$ (Lecture 17) $\nsubseteq$	(Lecture 16) $\longleftrightarrow$ (Lecture 11/12) CFG Chomsky Normal Form (Lecture 18)	(Lecture 11) CFL $\dots$ (Lecture 13) Parse Trees & Ambiguity Closure Properties (Lecture 19) Pumping Lemma (Lecture 20)
(Part 1) Finite Automata	(Lecture 4) NFA $\longleftrightarrow$ (Lecture 3) DFA $\longleftrightarrow$ (Lecture 5) ϵ -NFA $\longleftrightarrow$ (Lecture 7) RE Equivalence & Minimization (Lecture 10)	(Lecture 6)	(Lecture 3) RL $\dots$ Closure Properties (Lecture 8) Pumping Lemma (Lecture 9)
(Part 0) Basic Concepts	(Lecture 1) Mathematical Preliminaries	(Lecture 2) Scala	

The final exam will be given in class.

Date: 13:30-14:45 (1 hour 15 minutes), June 23 (Mon.).

Location: 301, Aegineung (애기능생활관 301호)

Coverage: Lectures 14 – 26

Format: 8 questions with closed book and closed notes

Yes/No questions about concepts in the theory of computation.

Construction of automata or grammars for given languages.

Proofs of given statements related to automata or grammars.

etc.

Note that there is **no class** on **June 18 (Wed.)**.

Please refer to the **previous exams** in the course website:

<https://plrg.korea.ac.kr/courses/cose215/>

I hope you enjoyed the class!

Jihyeok Park

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<https://plrg.korea.ac.kr>