Lecture 13 – Parse Trees and Ambiguity

COSE215: Theory of Computation

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PLRG

2026 Spring

Recall

- A **context-free grammar (CFG)**:

$$G = (V, \Sigma, S, R)$$

- The **language** of a CFG G:

$$L(G) = \{w \in \Sigma^* \mid S \Rightarrow^* w\}$$

- A language L is a **context-free language (CFL)**:

$$\exists \text{ CFG } G.\ L(G) = L$$

- For a given word $w \in L(G)$, a **derivation** for w is $S \Rightarrow^* w$
- A sequence $\alpha \in (V \cup \Sigma)^*$ is a **sentential form** if $S \Rightarrow^* \alpha$.

Contents

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Parse Trees

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$$S \to \epsilon \mid (S) \mid SS$$

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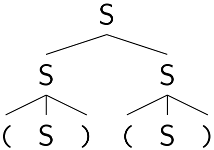
However, **parse trees** focus on the structure of the derivations instead of considering the order of the derivation steps.

For example, the above two derivations have the same parse tree:



Parse Trees

Definition (Parse Trees)

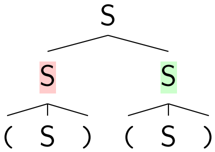
For a given CFG $G = (V, \Sigma, S, R)$, **parse trees** are trees satisfying:

1. The **root node** is labeled with the **start variable** S.
2. Each **internal node** is labeled with a **variable** $A \in V$.
   If its children are labeled with:
   $$X_1, X_2, \cdots, X_k$$
   from the left to the right, then $A \to X_1 X_2 \cdots X_k \in R$.
3. Each **leaf node** is labeled with a variable, symbol, or ϵ. However, if a leaf node is labeled with ϵ, it must be the only child of its parent.

Parse Trees – Example 1: Balanced Parentheses

$$S \rightarrow \epsilon \mid (S) \mid SS$$

A parse tree for $(S)(S)$:



$$① \quad S \Rightarrow_L S\,S \Rightarrow_L (S)S \Rightarrow (S)(S)$$

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Parse Trees – Example 2: Even Palindromes

$$S \rightarrow \epsilon \mid \mathtt{a}S\mathtt{a} \mid \mathtt{b}S\mathtt{b}$$

A parse tree for `abba`:

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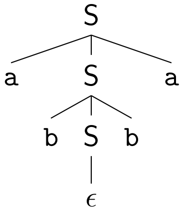

Parse Trees – Example 3: Arithmetic Expressions

$$
\begin{aligned}
S &\to N \mid X \mid S\texttt{+}S \mid S\texttt{*}S \mid \texttt{(}S\texttt{)} \\
N &\to D \mid DN \\
D &\to \texttt{0} \mid \cdots \mid \texttt{9} \\
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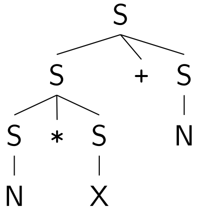
$$

A parse tree for $N\texttt{*}X\texttt{+}N$:



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The sequence obtained by concatenating the labels (without ϵ) of the leaf nodes of a parse tree from left to right is called the **yield** of the parse tree.

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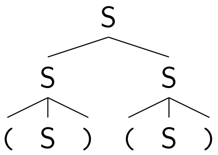

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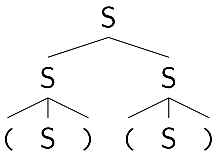

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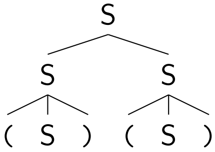

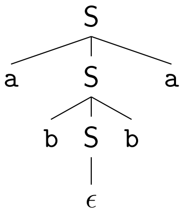

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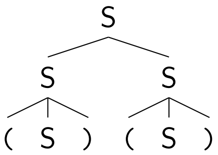

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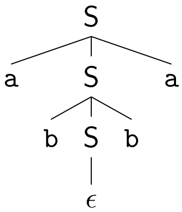

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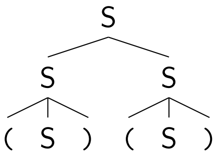

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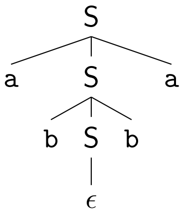

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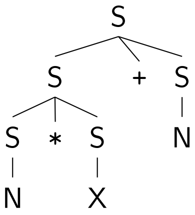

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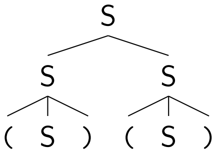

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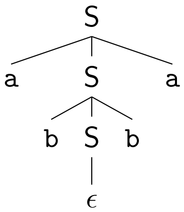

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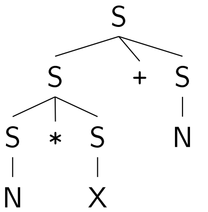

Its yield is $N{*}X{+}N$.

Relationship between Parse Trees and Derivations

Theorem (Parse Trees and Derivations)

For a given CFG $G = (V, \Sigma, S, R)$, *for any sequence* $\alpha \in (V \cup \Sigma)^*$*:*

$$S \Rightarrow^* \alpha \iff \exists \text{ parse tree } T. \text{ s.t. } T \text{ yields } \alpha$$

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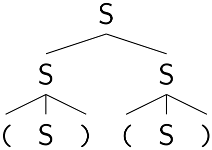
For example, consider the sequence $(S)(S)$:

$$S \Rightarrow SS \Rightarrow (S)S \Rightarrow (S)(S)$$

$$\Updownarrow$$



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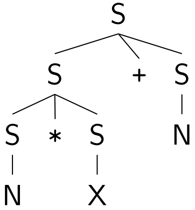

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For example, consider the sentential form $N\texttt{*}X\texttt{+}N$:

Actually, there are **two** parse trees for $N\texttt{*}X\texttt{+}N$.

Ambiguous Grammars

Definition (Ambiguous Grammar)

A context-free grammar $G = (V, \Sigma, S, R)$ is **ambiguous** if there exist two distinct parse trees for a word $w \in \Sigma^*$. If not, G is **unambiguous**.

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Theorem

Let $G = (V, \Sigma, S, R)$ be a CFG. Then, the following numbers are equal for any sequence of variables or symbols $w \in (V \cup \Sigma)^*$:

1. *The number of parse trees whose yields are w.*
2. *The number of left-most derivations for w.*
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Proof) We can convert a left-most (or right-most) derivation for a word w into the corresponding parse tree for w and vice versa.

Ambiguous Grammars – Example

$$
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S &\rightarrow N \mid X \mid S\texttt{+}S \mid S\texttt{*}S \mid \texttt{(}S\texttt{)} \\
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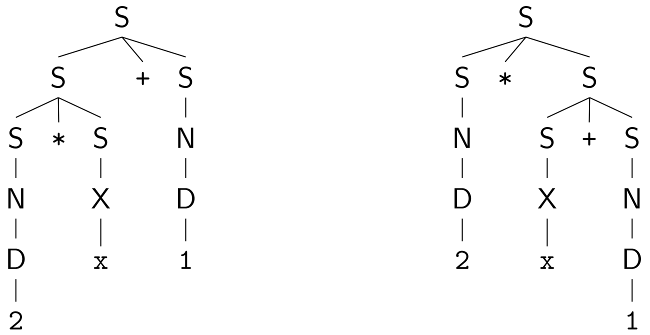
This grammar is **ambiguous** because there are **two** parse trees for the word `2 * x + 1`:



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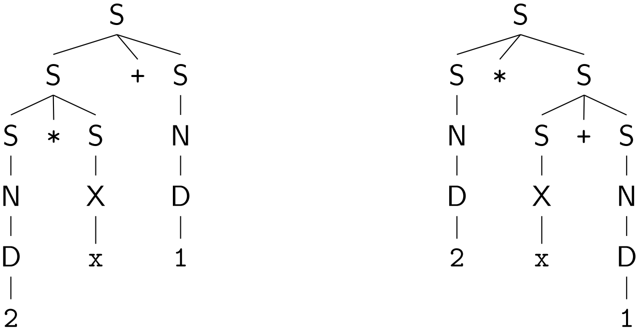

So, there are **two** left-most (or right-most) derivations for `2 * x + 1`.

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There are **two** left-most derivations for `2 * x + 1`:

1. Applying the production rule $S \rightarrow S\texttt{+}S$ first:

$$
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S &\Rightarrow_L S\texttt{+}S \Rightarrow_L S\texttt{*}S\texttt{+}S \Rightarrow_L N\texttt{*}S\texttt{+}S \Rightarrow_L D\texttt{*}S\texttt{+}S \Rightarrow_L \texttt{2*}S\texttt{+}S \\
&\Rightarrow_L \texttt{2*}X\texttt{+}S \Rightarrow_L \texttt{2*x+}S \Rightarrow_L \texttt{2*x+}N \Rightarrow_L \texttt{2*x+}D \Rightarrow_L \texttt{2*x+1}
\end{aligned}
$$

2. Applying the production rule $S \rightarrow S\texttt{*}S$ first:

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$$

For example, an equivalent but unambiguous grammar is:

$$
\begin{array}{rcl}
S & \rightarrow & T \mid S\texttt{+}T \\
T & \rightarrow & F \mid T\texttt{*}F \\
F & \rightarrow & N \mid X \mid \texttt{(}S\texttt{)} \\
N & \rightarrow & D \mid DN \\
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Now, the unique parse tree for `2 * x + 1` is:

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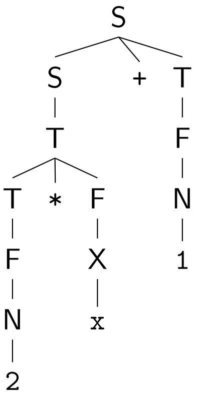

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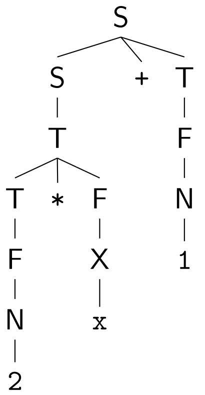

Let's try to understand how to eliminate the ambiguity in the original grammar.

Eliminating Ambiguity

First, analyze why the original grammar is ambiguous.

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- **Precedence** is not specified between different operators (`+` and `*`).
  - For example, two parse trees for `1 * 2 + 3` interpreted as:

    `1 * (2 + 3)` and `(1 * 2) + 3`

  - Let's give `*` higher precedence than `+` to interpret it as `(1 * 2) + 3`.

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  - Let's give * higher precedence than + to interpret it as `(1 * 2) + 3`.
- **Associativity** for the same operator (+ or *).
  - For example, two parse trees for `1 + 2 + 3` interpreted as:

    `1 + (2 + 3)` and `(1 + 2) + 3`

  - Let's give the left-associativity to + to interpret it as `(1 + 2) + 3`.

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To enforce the **precedence**, define new variables F for factors and T for terms:

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- A **factor** is a number, a variable, or a parenthesized expression:

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- A **term** is the multiplication of one or more factors:

  $$\texttt{42}, \quad \texttt{2 * x}, \quad \texttt{2 * (1 + 2)}, \quad \texttt{1 * (x * y) * z}, \quad \cdots$$

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  $$T \rightarrow F \mid T * F$$

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  `42,   2 * x,   2 * (1 + 2),   1 * (x * y) * z,   ⋯`

  In the grammar, T is defined as:

  $$T \to F \mid T{*}F$$

- An **expression** is the addition of one or more terms:

  `42,   1 + 2,   1 + 2 * 3,   (1 + 2) * 3 + 4,   ⋯`

  In the grammar, S is defined as:

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Eliminating Ambiguity – Associativity

The unambiguous grammar is:

$$
\begin{array}{rcl}
S & \rightarrow & T \mid S\texttt{+}T \\
T & \rightarrow & F \mid T\texttt{*}F \\
F & \rightarrow & N \mid X \mid \texttt{(}S\texttt{)} \\
N & \rightarrow & D \mid DN \\
D & \rightarrow & \texttt{0} \mid \cdots \mid \texttt{9} \\
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- This grammar supports the **left-associativity** of + and $*$. Why?
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- This grammar supports the **left-associativity** of + and $*$. Why?
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- Then, how to support the **right-associativity** of + and $*$?
  - Replace the **left-recursive** rules with **right-recursive** rules!

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T &\to F \mid F*T \\
&\cdots
\end{aligned}
$$

So far, we have discussed the **ambiguity** for **grammars**.
We will now discuss the **inherent ambiguity** for **languages**.

Definition (Inherent Ambiguity)

A language L is **inherently ambiguous** if all CFGs whose languages are L are ambiguous. (i.e. there is no unambiguous grammar for L)

[1] `https://en.wikipedia.org/wiki/Ogden's_lemma`

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For example, the following language is **inherently ambiguous**:

$$L = \{\mathrm{a}^i \mathrm{b}^j \mathrm{c}^k \mid i, j, k \geq 0 \land (i = j \lor j = k)\}$$

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$$\begin{array}{llll} S \rightarrow L \mid R & L \rightarrow X \mid L\mathtt{c} & R \rightarrow Y \mid \mathtt{a}R \\ & X \rightarrow \epsilon \mid \mathtt{a}X\mathtt{b} & Y \rightarrow \epsilon \mid \mathtt{b}Y\mathtt{c} \end{array}$$

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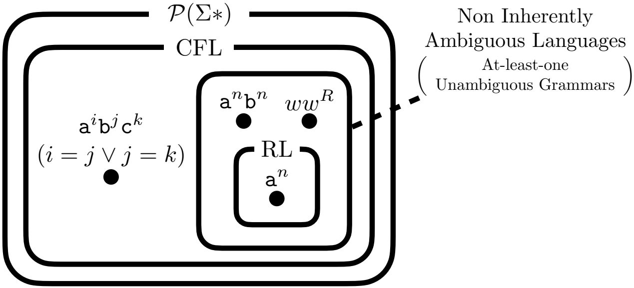
While we can prove that L is inherently ambiguous using the Ogden's lemma[1], we will not discuss it in this course.

[1] https://en.wikipedia.org/wiki/Ogden's_lemma

Summary

1. Parse Trees
   - Definition
   - Yields
   - Relationship between Parse Trees and Derivations
2. Ambiguity
   - Ambiguous Grammars
   - Eliminating Ambiguity
   - Inherent Ambiguity



Midterm Exam

- The midterm exam will be given in class.
- **Date:** 13:30-14:45 (1 hour 15 minutes), April 22 (Wed.).
- **Location:**
  - **IT-Education Hall B102:** Students with Student ID 2025XXXX
  - **IT-Education Hall 611:** All other students
- **Coverage:** Lectures 1 – 13 (No coding question)
- **Format:** 7–9 questions with closed book and closed notes
- Please refer to the **previous exams** in the course website:

  https://plrg.korea.ac.kr/courses/cose215/

- **Apr. 27 (Mon.):** No class
- **Apr. 29 (Wed.):** Recorded video lecture (Online).

Next Lecture

- Pushdown Automata (PDA)

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