

Lecture 27 – Course Review

COSE215: Theory of Computation

Jihyeok Park



2026 Spring

- What is the *mathematical model* of computers?

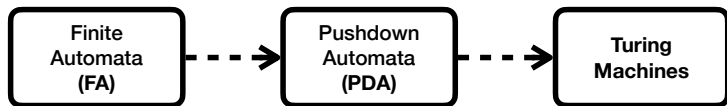
Turing Machine!

Let's learn **Turing Machine**

- Is it possible to solve *every problem* using computers?

No!

Let's learn **Undecidability** and **Intractability**



- **Part 1: Finite Automata (FA)**
 - Regular Expressions (REs)
 - Regular Languages (RLs)
 - Applications: text search, etc.
- **Part 2: Pushdown Automata (PDA)**
 - Context-Free Grammars (CFGs)
 - Context-Free Languages (CFLs)
 - Applications: programming languages, natural language processing, etc.
- **Part 3: Turing Machines (TMs)**
 - Lambda Calculus (LC)
 - Recursively Enumerable Languages (REs)
 - Undecidability and Intractability

	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata			
(Part 1) Finite Automata			
(Part 0) Basic Concepts			

	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata			
(Part 1) Finite Automata			
(Part 0) Basic Concepts	(Lecture 1) Mathematical Preliminaries		

	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata			
(Part 1) Finite Automata			
(Part 0) Basic Concepts	(Lecture 1) Mathematical Preliminaries	(Lecture 2) Scala	

	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata			
(Part 1) Finite Automata	(Lecture 3) DFA		(Lecture 3) RL
(Part 0) Basic Concepts	(Lecture 1) Mathematical Preliminaries	(Lecture 2) Scala	

	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata			
(Part 1) Finite Automata	(Lecture 4) NFA \longleftrightarrow (Lecture 3) DFA		(Lecture 3) RL
(Part 0) Basic Concepts	(Lecture 1) Mathematical Preliminaries	(Lecture 2) Scala	

	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata			
(Part 1) Finite Automata	(Lecture 4) NFA \longleftrightarrow (Lecture 3) DFA \longleftrightarrow (Lecture 5) ϵ -NFA		(Lecture 3) RL
(Part 0) Basic Concepts	(Lecture 1) Mathematical Preliminaries	(Lecture 2) Scala	

	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata			
(Part 1) Finite Automata	(Lecture 4) (Lecture 3) (Lecture 5) NFA \longleftrightarrow DFA \longleftrightarrow ϵ -NFA	(Lecture 6) RE	(Lecture 3) RL
(Part 0) Basic Concepts	(Lecture 1) Mathematical Preliminaries	(Lecture 2) Scala	

	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata			
(Part 1) Finite Automata	(Lecture 4) NFA	(Lecture 3) DFA	(Lecture 3) RL
(Part 0) Basic Concepts	(Lecture 1) Mathematical Preliminaries	(Lecture 2) Scala	

(Lecture 5) ϵ -NFA

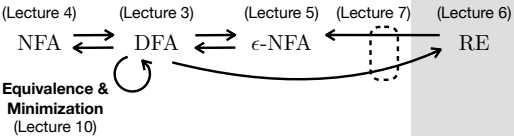
 (Lecture 7)

 (Lecture 6) RE

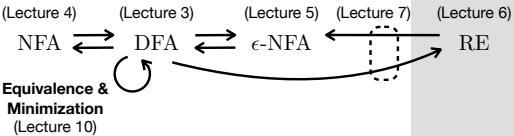
Bidirectional arrows connect NFA and DFA. Bidirectional arrows connect DFA and ϵ -NFA. A dashed box encloses ϵ -NFA and RE, with a curved arrow pointing from ϵ -NFA to RE.

	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata			
(Part 1) Finite Automata	(Lecture 4) NFA	(Lecture 3) DFA	(Lecture 5) ϵ -NFA
	↔	↔	↔
			(Lecture 7) RE
			(Lecture 6) RE
			(Lecture 3) RL ⋮ Closure Properties (Lecture 8)
(Part 0) Basic Concepts	(Lecture 1) Mathematical Preliminaries		(Lecture 2) Scala

	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata			
(Part 1) Finite Automata	(Lecture 4) NFA	(Lecture 3) DFA	(Lecture 5) ϵ -NFA
	↔	↔	↔
			(Lecture 7) RE
			(Lecture 6) ↖ (dashed arrow) ↗ (solid arrow)
			(Lecture 3) RL Closure Properties (Lecture 8)
			Pumping Lemma (Lecture 9)
(Part 0) Basic Concepts	(Lecture 1) Mathematical Preliminaries	(Lecture 2) Scala	

	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata			
(Part 1) Finite Automata	<p>(Lecture 4) NFA \rightleftarrows DFA \rightleftarrows ϵ-NFA (Lecture 3)</p> <p>Equivalence & Minimization (Lecture 10)</p> 	<p>(Lecture 6) RE</p> <p>(Lecture 3) RL</p> <p>Closure Properties (Lecture 8)</p> <p>Pumping Lemma (Lecture 9)</p>	
(Part 0) Basic Concepts	(Lecture 1) Mathematical Preliminaries	(Lecture 2) Scala	

	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata		(Lecture 11/12) CFG	(Lecture 11) CFL
(Part 1) Finite Automata	(Lecture 4) NFA \leftrightarrow DFA \leftrightarrow ϵ -NFA Equivalence & Minimization (Lecture 10)	(Lecture 5) ϵ -NFA \leftrightarrow RE (Lecture 7) RE \leftrightarrow DFA	(Lecture 3) RL Closure Properties (Lecture 8) Pumping Lemma (Lecture 9)
(Part 0) Basic Concepts	(Lecture 1) Mathematical Preliminaries	(Lecture 2) Scala	

	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata		(Lecture 11/12) CFG	(Lecture 11) CFL ... (Lecture 13) Parse Trees & Ambiguity
(Part 1) Finite Automata	(Lecture 4) NFA \rightleftarrows (Lecture 3) DFA \rightleftarrows (Lecture 5) ϵ -NFA \leftarrow (Lecture 7) RE \rightarrow (Lecture 6) RE Equivalence & Minimization (Lecture 10) 		(Lecture 3) RL Closure Properties (Lecture 8) ... Pumping Lemma (Lecture 9)
(Part 0) Basic Concepts	(Lecture 1) Mathematical Preliminaries	(Lecture 2) Scala	

	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata	(Lecture 14/15) PDA _{FS} PDA _{ES}	(Lecture 11/12) CFG	(Lecture 11) CFL ... (Lecture 13) Parse Trees & Ambiguity
(Part 1) Finite Automata	(Lecture 4) NFA ↔ (Lecture 3) DFA ↔ (Lecture 5) ϵ -NFA ↔ (Lecture 7) RE Equivalence & Minimization (Lecture 10)	(Lecture 6) RE	(Lecture 3) RL Closure Properties (Lecture 8) Pumping Lemma (Lecture 9)
(Part 0) Basic Concepts	(Lecture 1) Mathematical Preliminaries	(Lecture 2) Scala	

	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata	(Lecture 14/15) $PDA_{FS} \rightleftarrows PDA_{ES}$	(Lecture 16) \rightleftarrows	(Lecture 11/12) CFG (Lecture 11) CFL ... (Lecture 13) Parse Trees & Ambiguity
(Part 1) Finite Automata	(Lecture 4) NFA \rightleftarrows (Lecture 3) DFA (Lecture 5) \rightleftarrows ϵ -NFA (Lecture 7) \rightleftarrows (Lecture 6) RE Equivalence & Minimization (Lecture 10)		(Lecture 3) RL Closure Properties (Lecture 8) Pumping Lemma (Lecture 9)
(Part 0) Basic Concepts	(Lecture 1) Mathematical Preliminaries	(Lecture 2) Scala	

	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata	<p>(Lecture 14/15)</p> $\text{PDA}_{\text{FS}} \begin{array}{c} \xleftrightarrow{\text{---}} \\ \xleftrightarrow{\text{---}} \\ \xleftrightarrow{\text{---}} \end{array} \text{PDA}_{\text{ES}}$ <p>\cup</p> $\text{DPDA}_{\text{FS}} \supset \text{DPDA}_{\text{ES}}$ <p>\cup (Lecture 17) $\not\subset$</p>	<p>(Lecture 16)</p> $\text{PDA}_{\text{ES}} \begin{array}{c} \xleftrightarrow{\text{---}} \\ \xleftrightarrow{\text{---}} \end{array} \text{CFG}$	<p>(Lecture 11/12)</p> <p>(Lecture 11) CFL ... (Lecture 13) Parse Trees & Ambiguity</p>
(Part 1) Finite Automata	<p>(Lecture 4)</p> $\text{NFA} \begin{array}{c} \xleftrightarrow{\text{---}} \\ \xleftrightarrow{\text{---}} \end{array} \text{DFA}$ <p>Equivalence & Minimization (Lecture 10)</p>	<p>(Lecture 3)</p> $\text{DFA} \begin{array}{c} \xleftrightarrow{\text{---}} \\ \xleftrightarrow{\text{---}} \end{array} \epsilon\text{-NFA}$ <p>(Lecture 5)</p> $\epsilon\text{-NFA} \begin{array}{c} \xleftrightarrow{\text{---}} \\ \xleftrightarrow{\text{---}} \end{array} \text{RE}$ <p>(Lecture 7)</p> <p>(Lecture 6)</p>	<p>(Lecture 3)</p> RL <p>⋮</p> <p>Closure Properties (Lecture 8) Pumping Lemma (Lecture 9)</p>
(Part 0) Basic Concepts	<p>(Lecture 1)</p> <p>Mathematical Preliminaries</p>	<p>(Lecture 2)</p> <p>Scala</p>	

	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata	<p>(Lecture 14/15)</p> $\text{PDA}_{\text{FS}} \begin{array}{c} \xleftrightarrow{\text{---}} \\ \xleftrightarrow{\text{---}} \\ \xleftrightarrow{\text{---}} \end{array} \text{PDA}_{\text{ES}}$ <p>\cup</p> $\text{DPDA}_{\text{FS}} \supset \text{DPDA}_{\text{ES}}$ <p>\cup (Lecture 17) $\not\subset$</p>	<p>(Lecture 16)</p> $\text{PDA}_{\text{ES}} \xleftrightarrow{\text{---}} \text{CFG}$ <p>(Lecture 11/12)</p> \vdots <p>Chomsky Normal Form (Lecture 18)</p>	<p>(Lecture 11)</p> $\text{CFL} \dots$ <p>(Lecture 13) Parse Trees & Ambiguity</p>
(Part 1) Finite Automata	<p>(Lecture 4)</p> $\text{NFA} \xleftrightarrow{\text{---}} \text{DFA}$ <p>(Lecture 3)</p> $\text{DFA} \xleftrightarrow{\text{---}} \epsilon\text{-NFA}$ <p>(Lecture 5)</p> $\epsilon\text{-NFA} \xleftrightarrow{\text{---}} \text{RE}$ <p>(Lecture 7)</p> <p>(Lecture 6)</p> <p>Equivalence & Minimization (Lecture 10)</p>		<p>(Lecture 3)</p> RL <p>\vdots</p> <p>Closure Properties (Lecture 8)</p> <p>Pumping Lemma (Lecture 9)</p>
(Part 0) Basic Concepts	<p>(Lecture 1)</p> <p>Mathematical Preliminaries</p>	<p>(Lecture 2)</p> <p>Scala</p>	

	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata	<p>(Lecture 14/15)</p> $\text{PDA}_{\text{FS}} \begin{array}{c} \xleftrightarrow{\text{---}} \\ \xleftrightarrow{\text{---}} \\ \xleftrightarrow{\text{---}} \end{array} \text{PDA}_{\text{ES}}$ <p>\cup</p> $\text{DPDA}_{\text{FS}} \supset \text{DPDA}_{\text{ES}}$ <p>\cup (Lecture 17) $\not\subset$</p>	<p>(Lecture 16)</p> $\text{PDA}_{\text{ES}} \begin{array}{c} \xleftrightarrow{\text{---}} \\ \xleftrightarrow{\text{---}} \end{array} \text{CFG}$ <p>(Lecture 11/12)</p> <p>⋮</p> <p>Chomsky Normal Form (Lecture 18)</p>	<p>(Lecture 11)</p> <p>CFL ... Parse Trees & Ambiguity</p> <p>⋮</p> <p>Closure Properties (Lecture 19)</p>
(Part 1) Finite Automata	<p>(Lecture 4)</p> <p>NFA $\xleftrightarrow{\text{---}}$ DFA $\xleftrightarrow{\text{---}}$ ϵ-NFA</p> <p>(Lecture 3)</p> <p>(Lecture 5)</p> <p>(Lecture 7)</p> <p>(Lecture 6)</p> <p>RE</p> <p>Equivalence & Minimization (Lecture 10)</p>		<p>(Lecture 3)</p> <p>RL</p> <p>⋮</p> <p>Closure Properties (Lecture 8)</p> <p>Pumping Lemma (Lecture 9)</p>
(Part 0) Basic Concepts	<p>(Lecture 1)</p> <p>Mathematical Preliminaries</p>	<p>(Lecture 2)</p> <p>Scala</p>	

	Automata	Grammars	Languages
(Part 3) Turing Machines			
(Part 2) Pushdown Automata	<p>(Lecture 14/15)</p> $\text{PDA}_{\text{FS}} \begin{matrix} \xrightarrow{\text{dashed}} \\ \xleftrightarrow{\text{dashed}} \\ \xleftarrow{\text{dashed}} \end{matrix} \text{PDA}_{\text{ES}}$ <p>\cup</p> $\text{DPDA}_{\text{FS}} \supset \text{DPDA}_{\text{ES}}$ <p>\cup (Lecture 17) $\not\subseteq$</p>	<p>(Lecture 16)</p> $\text{PDA}_{\text{ES}} \xleftrightarrow{\text{dashed}} \text{CFG}$ <p>(Lecture 11/12)</p> <p>CFG</p> <p>⋮</p> <p>Chomsky Normal Form (Lecture 18)</p>	<p>(Lecture 11)</p> <p>CFL</p> <p>⋮</p> <p>Closure Properties (Lecture 19)</p> <p>(Lecture 13)</p> <p>Parse Trees & Ambiguity</p> <p>⋮</p> <p>Pumping Lemma (Lecture 20)</p>
(Part 1) Finite Automata	<p>(Lecture 4)</p> <p>NFA</p> <p>↔</p> <p>(Lecture 3)</p> <p>DFA</p> <p>↔</p> <p>(Lecture 5)</p> <p>ϵ-NFA</p> <p>↔</p> <p>(Lecture 7)</p> <p>RE</p> <p>(Lecture 6)</p> <p>↔</p> <p>Equivalence & Minimization (Lecture 10)</p>		<p>(Lecture 3)</p> <p>RL</p> <p>⋮</p> <p>Closure Properties (Lecture 8)</p> <p>⋮</p> <p>Pumping Lemma (Lecture 9)</p>
(Part 0) Basic Concepts	<p>(Lecture 1)</p> <p>Mathematical Preliminaries</p>	<p>(Lecture 2)</p> <p>Scala</p>	

	Automata	Grammars	Languages
(Part 3) Turing Machines	(Lecture 21/22) TM		(Lecture 21) REL
(Part 2) Pushdown Automata	(Lecture 14/15) $PDA_{FS} \leftrightarrow PDA_{ES}$ \cup $DPDA_{FS} \supset DPDA_{ES}$ \cup (Lecture 17) \cup	(Lecture 16) \leftrightarrow (Lecture 11/12) CFG ⋮ Chomsky Normal Form (Lecture 18)	(Lecture 11) CFL ⋮ Closure Properties (Lecture 19) (Lecture 13) Parse Trees & Ambiguity ⋮ Pumping Lemma (Lecture 20)
(Part 1) Finite Automata	(Lecture 4) NFA \leftrightarrow (Lecture 3) DFA \leftrightarrow (Lecture 5) ϵ -NFA \leftrightarrow (Lecture 7) RE Equivalence & Minimization (Lecture 10)	(Lecture 6)	(Lecture 3) RL ⋮ Closure Properties (Lecture 8) Pumping Lemma (Lecture 9)
(Part 0) Basic Concepts	(Lecture 1) Mathematical Preliminaries	(Lecture 2) Scala	

	Automata	Grammars	Languages
(Part 3) Turing Machines	(Lecture 23) ETM \longleftrightarrow (Lecture 21/22) TM		(Lecture 21) REL
(Part 2) Pushdown Automata	(Lecture 14/15) PDA _{FS} \longleftrightarrow PDA _{ES} \cup DPDA _{FS} \supset DPDA _{ES} \cup (Lecture 17) \curvearrowright	(Lecture 16) \longleftrightarrow (Lecture 11/12) CFG ⋮ Chomsky Normal Form (Lecture 18)	(Lecture 11) CFL ⋮ Closure Properties (Lecture 19) (Lecture 13) Parse Trees & Ambiguity ⋮ Pumping Lemma (Lecture 20)
(Part 1) Finite Automata	(Lecture 4) NFA \longleftrightarrow (Lecture 3) DFA \longleftrightarrow (Lecture 5) ϵ -NFA \longleftrightarrow (Lecture 7) RE Equivalence & Minimization (Lecture 10)	(Lecture 6)	(Lecture 3) RL ⋮ Closure Properties (Lecture 8) (Lecture 9) Pumping Lemma
(Part 0) Basic Concepts	(Lecture 1) Mathematical Preliminaries	(Lecture 2) Scala	

	Automata	Grammars	Languages
(Part 3) Turing Machines	(Lecture 23) $ETM \rightleftharpoons TM$	(Lecture 21/22) $TM \rightleftharpoons LC$ (Lecture 24)	(Lecture 21) REL
(Part 2) Pushdown Automata	(Lecture 14/15) $PDA_{FS} \rightleftharpoons PDA_{ES}$ \cup $DPDA_{FS} \supset DPDA_{ES}$ \cup (Lecture 17) \cup	(Lecture 16) $PDA_{ES} \rightleftharpoons CFG$ (Lecture 11/12) Chomsky Normal Form (Lecture 18)	(Lecture 11) CFL (Lecture 13) Parse Trees & Ambiguity (Lecture 19) Closure Properties (Lecture 20) Pumping Lemma
(Part 1) Finite Automata	(Lecture 4) $NFA \rightleftharpoons DFA$ (Lecture 3) $DFA \rightleftharpoons \epsilon\text{-NFA}$ (Lecture 5) $\epsilon\text{-NFA} \rightleftharpoons RE$ (Lecture 7) $RE \rightleftharpoons DFA$ (Lecture 6)		(Lecture 3) RL (Lecture 8) Closure Properties (Lecture 9) Pumping Lemma
(Part 0) Basic Concepts	(Lecture 1) Mathematical Preliminaries	(Lecture 2) Scala	

	Automata	Grammars	Languages
(Part 3) Turing Machines	(Lecture 23) $ETM \rightleftharpoons TM$	(Lecture 21/22) $TM \rightleftharpoons LC$	(Lecture 24) REL \cup DL (Lecture 25)
(Part 2) Pushdown Automata	(Lecture 14/15) $PDA_{FS} \rightleftharpoons PDA_{ES}$ \cup $DPDA_{FS} \supset DPDA_{ES}$ \cup (Lecture 17)	(Lecture 16) $PDA_{ES} \rightleftharpoons CFG$ \vdots Chomsky Normal Form (Lecture 18)	(Lecture 11) CFL \vdots Closure Properties (Lecture 19) (Lecture 13) Parse Trees & Ambiguity \vdots Pumping Lemma (Lecture 20)
(Part 1) Finite Automata	(Lecture 4) NFA \rightleftharpoons (Lecture 3) DFA Equivalence & Minimization (Lecture 10)	(Lecture 5) ϵ -NFA \rightleftharpoons (Lecture 7) RE (Lecture 6)	(Lecture 3) RL \vdots Closure Properties (Lecture 8) Pumping Lemma (Lecture 9)
(Part 0) Basic Concepts	(Lecture 1) Mathematical Preliminaries	(Lecture 2) Scala	

	Automata	Grammars	Languages
(Part 3) Turing Machines	(Lecture 23) $\text{ETM} \rightleftharpoons \text{TM}$ (Lecture 21/22)	(Lecture 24) $\text{TM} \rightleftharpoons \text{LC}$	(Lecture 21) REL (Lecture 26) \cup $\text{DL} \supset \text{NP} \stackrel{?}{=} \text{P}$ (Lecture 25) \supset
(Part 2) Pushdown Automata	(Lecture 14/15) $\text{PDA}_{\text{FS}} \rightleftharpoons \text{PDA}_{\text{ES}}$ \cup $\text{DPDA}_{\text{FS}} \supset \text{DPDA}_{\text{ES}}$ \cup (Lecture 17) $\not\subseteq$	(Lecture 16) $\text{PDA}_{\text{ES}} \rightleftharpoons \text{CFG}$ (Lecture 11/12) \vdots Chomsky Normal Form (Lecture 18)	(Lecture 11) CFL (Lecture 13) Parse Trees & Ambiguity \vdots Closure Properties (Lecture 19) Pumping Lemma (Lecture 20)
(Part 1) Finite Automata	(Lecture 4) $\text{NFA} \rightleftharpoons \text{DFA}$ (Lecture 3) (Lecture 5) $\text{DFA} \rightleftharpoons \epsilon\text{-NFA}$ (Lecture 7) (Lecture 6) $\epsilon\text{-NFA} \rightleftharpoons \text{RE}$ (Lecture 10) Equivalence & Minimization		(Lecture 3) RL \vdots Closure Properties (Lecture 8) Pumping Lemma (Lecture 9)
(Part 0) Basic Concepts	(Lecture 1) Mathematical Preliminaries	(Lecture 2) Scala	

- The final exam will be given in class.
- **Date:** 13:30-14:45 (1 hour 15 minutes), June 17 (Wed.).
- **Location:**
 - **IT-Education Hall B102:** Students with Student ID 2025XXXX
 - **IT-Education Hall 611:** All other students
- **Coverage:** Lectures 14 – 26
- **Format:** 7–9 questions with closed book and closed notes
 - Yes/No questions about concepts in the theory of computation.
 - Construction of automata or grammars for given languages.
 - Proofs of given statements related to automata or grammars.
 - etc.
- Note that there is **no class** on **June 22 (Mon.)**.
- Please refer to the **previous exams** in the course website:

<https://plrg.korea.ac.kr/courses/cose215/>

- I hope you enjoyed the class!

Jihyeok Park

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`https://plrg.korea.ac.kr`