Midterm Exam

COSE215: Theory of Computation
2026 Spring

Instructor: Jihyeok Park

April 22, 2026. 13:30-14:45

- **If you are not good at English, please write your answers in Korean.**
  (영어가 익숙하지 않은 경우, 답안을 한글로 작성해 주세요.)
- **Write answers in good handwriting.**
  If we cannot recognize your answers, you will not get any points.
  (글씨를 알아보기 힘들면 점수를 드릴 수 없습니다. 답안을 읽기 좋게 작성해주세요.)
- **Write your answers in the boxes provided.**
  (답안을 제공된 박스 안에 작성해 주세요.)

Student ID	
Student Name	

Question:	1	2	3	4	5	6	7	8	Total
Points:	15	20	5	15	15	12	8	10	100
Score:									

1. [15 points] Design **deterministic finite automata (DFA)** using **transition diagrams** that accept the following languages.

   (a) [5 points] [☆☆☆] $L = \{w \in \{\mathtt{0}, \mathtt{1}\}^* \mid \text{every odd position of } w \text{ is } \mathtt{1}\}$. For example, $\epsilon, \mathtt{111010101} \in L$.

   (b) [5 points] [★☆☆] $L = \{w \in \{\mathtt{a}, \mathtt{b}\}^+ \mid \mathsf{first}(w) \neq \mathsf{last}(w)\}$ where $\mathsf{first}(w)$ and $\mathsf{last}(w)$ are the first and last symbols of w, respectively. For example, $\mathtt{abb} \in L$ and $\mathtt{abba} \notin L$.

   (c) [5 points] [★★☆] $L \subseteq \{\mathtt{0}, \mathtt{1}, \mathtt{2}\}^*$ such that $h(L) = \{\mathtt{a}^n \mid n \not\equiv 0 \pmod 3\}$ where h is a homomorphism defined as $h(i) = \mathtt{a}^i$ for $i = 0, 1, 2$.

2. 20 points Write **regular expressions (REs)** that represent the following languages.

(a) 5 points [☆☆☆] $L = \{\mathtt{a}^n\mathtt{b}^m \mid n + m \equiv 1 \pmod 2\}$.

$R =$

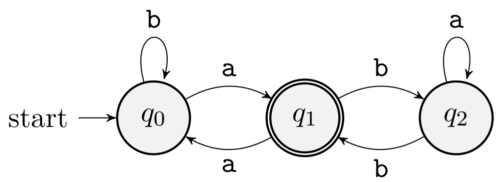
(b) 5 points [☆☆☆] $L = \{w \in \{\mathtt{a}, \mathtt{b}\}^* \mid$ every a in w is followed by at least two b's$\}$. For example, babb $\in L$.

$R =$

(c) 5 points [★★☆] $L = h(L')$ where h is a homomorphism defined as $h(\mathtt{0}) = \mathtt{a}$ and $h(\mathtt{1}) = \mathtt{ba}$, and L' is the language of the regular expression `(01)*1`.

$R =$

(d) 5 points [★★☆] L is the language of the following DFA:



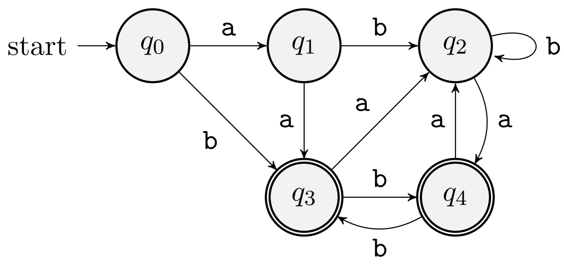
$R =$

3. 5 points [★★★] Design an **ϵ-nondeterministic finite automaton (ϵ-NFA)** using **transition diagrams** that accepts the following language with **at most six states**:

$$L = \{w \in \{\mathtt{0}, \mathtt{1}\}^* \mid w \text{ is obtained by deleting exactly one symbol from some word in } \mathtt{(010)}^*\}$$

4. 15 points Let's **minimize** the number of states of the following DFA $D = (Q, \Sigma, \delta, q_0, F)$:

$D =$



(a) 5 points [★☆☆] Fill in the table to represent the **distinguishable** pairs of states $Q \times Q$ of D:

q_1				
q_2				
q_3				
q_4				
	q_0	q_1	q_2	q_3

(b) 5 points [★☆☆] Define the **equivalence classes** $Q/_{\equiv}$ of the states in D using the result of the previous question, and fill in the **transition table** of the **minimized DFA** $D/_{\equiv} = (Q/_{\equiv}, \Sigma, \delta/_{\equiv}, p_0, F/_{\equiv})$:

$D/_{\equiv} =$

	$Q/_{\equiv}$	a	b
	$p_0 = \{$ $\}$		
	$p_1 = \{$ $\}$		
	$p_2 = \{$ $\}$		

(c) 5 points [★★☆] Consider another DFA $D' = (Q', \Sigma, \delta', q'_0, F')$ where $L(D') = L(D/_{\equiv})$.
Then, the following fact always holds:

$$\forall q \in Q,\ \exists q' \in Q'.\ q' \equiv q$$

Using the above fact, **prove** that $D/_{\equiv}$ always has **less than or equal** number of states than D':

5. 15 points Prove that the following language is **NOT regular** in two different ways:

$$L = \{\mathtt{a}^n\mathtt{b}^m \mid n, m \geq 0 \land n \neq m\}$$

(a) 6 points [★☆☆] Prove that $L = \{\mathtt{a}^n\mathtt{b}^m \mid n, m \geq 0 \land n \neq m\}$ is **NOT regular** using the **closure properties** of regular languages and the fact that $\{\mathtt{a}^n\mathtt{b}^n \mid n \geq 0\}$ is **NOT regular**:

(b) 9 points [★★☆] Fill in the blanks in the **proofs** to show that $L = \{\mathtt{a}^n\mathtt{b}^m \mid n, m \geq 0 \land n \neq m\}$ is **NOT regular** using the **pumping lemma** for regular languages:

1. Assume that any positive integer n is given. (i.e., $n \geq 1$)
2. Pick a word $w = \mathtt{a}^n\mathtt{b}^{n+n!} \in L$.
3. $|w| =$ ______ $\geq n$.
4. Assume that any split $w = xyz$ satisfying ① $|y| > 0$ and ② $|xy| \leq n$ is given.
5. We need to show that ¬③ $xy^iz \notin L$ for some $i \geq 0$.

6. 12 points Design **context-free grammars (CFGs)** that represent the following languages.

   (a) 5 points [★☆☆] $L = \{\texttt{a}^n\texttt{b}^m \mid n \leq m \leq 2n\}$

   (b) 7 points [★★☆] $L = \{w \in \{\texttt{a}, \texttt{b}\}^* \mid N_{\texttt{b}}(w) \leq N_{\texttt{a}}(w)\}$.

   Note that $N_{\texttt{a}}(w)$ and $N_{\texttt{b}}(w)$ are the number of a's and b's in w, respectively.

7. 8 points Consider the following an **ambiguous CFG** G:

$$S \rightarrow S\texttt{+}S \mid S\texttt{-}S \mid \texttt{(}S\texttt{)} \mid \texttt{x}$$

   (a) 3 points [☆☆☆] Pick a word $w \in L(G)$ that has two distinct parse trees and draw them of w:

   $w =$

   tree 1:

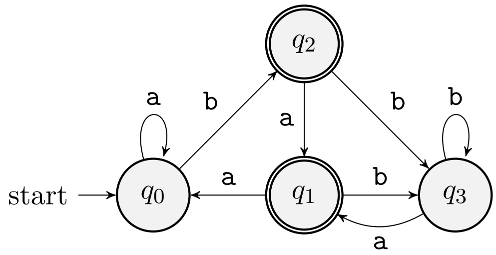
   tree 2:

   (b) 5 points [★☆☆] Convert G into an equivalent **unambiguous CFG** having a higher **precedence** of + than - and **right-associativity** of both + and -:

8. 10 points For a given DFA $D = (Q = \{q_0, q_1, \ldots, q_n\}, \Sigma, \delta, q_0, F)$, we can convert D into an equivalent CFG $G = (V = \{A_0, A_1, \ldots, A_n\}, \Sigma, R, A_0)$ that satisfies the following conditions:

$$\forall 0 \leq i \leq n,\ \forall w \in \Sigma^*.\ A_i \Rightarrow^* w \text{ if and only if } \delta^*(q_i, w) \in F$$

(a) 3 points [★★☆] Convert the following DFA D into an equivalent CFG G satisfying the above conditions:



(b) 7 points [★★★] Explain a **general procedure** to convert any DFA into an equivalent CFG satisfying the above conditions, and **discuss** whether the resulting CFG is **ambiguous** or not in general.

This is the last page.
I hope that your tests went well!