Construct a **Turing machine** that **computes** the function:

$$f(w \in \{0,1\}^*) = w + 1 \qquad \text{where} \qquad w \text{ starts with } 1$$

w =	$\cdots$	B	1	0	1	0	1	1	1	1	B	$\cdots$
$f(w)$ =	$\cdots$	B	1	0	1	1	0	0	0	0	B	$\cdots$

1: Check if the first bit is 1.
2: Move to the end of the input.
3: Repeatedly replace the rightmost 1 with 0.
4: Replace 0 (or B) with 1.
5: Go to the first input symbol.

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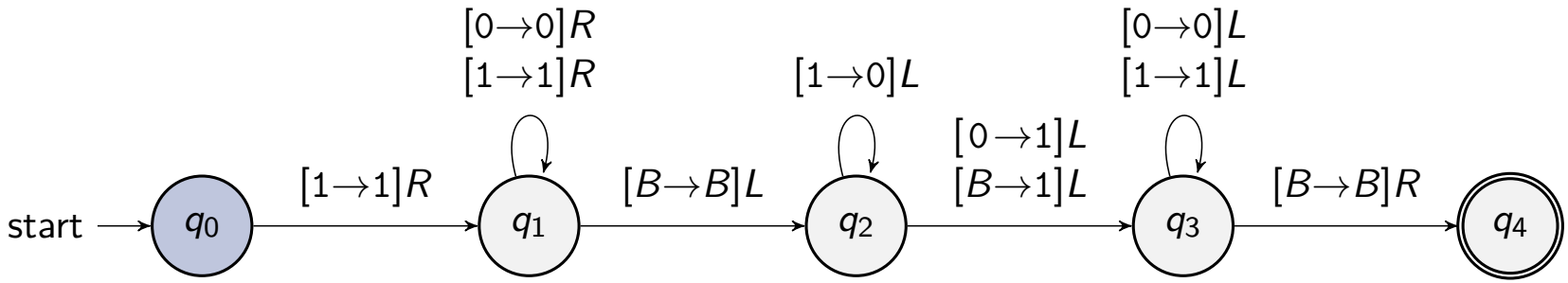

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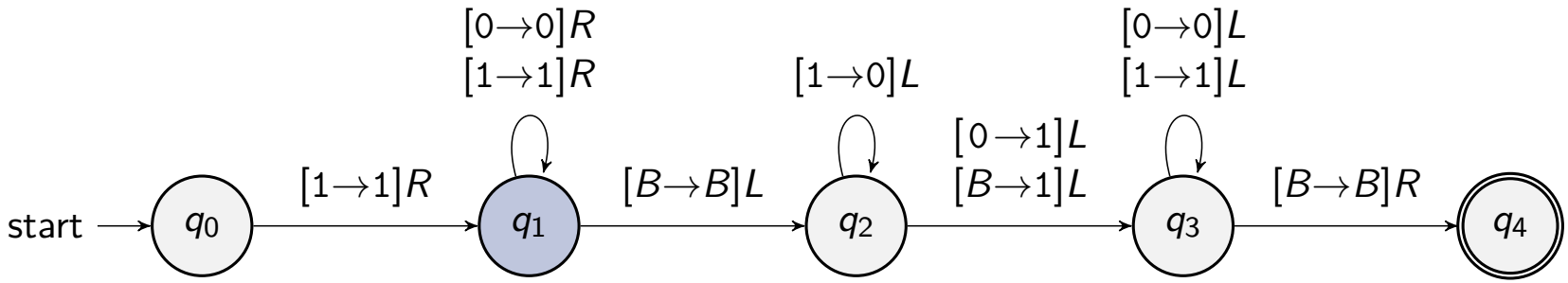

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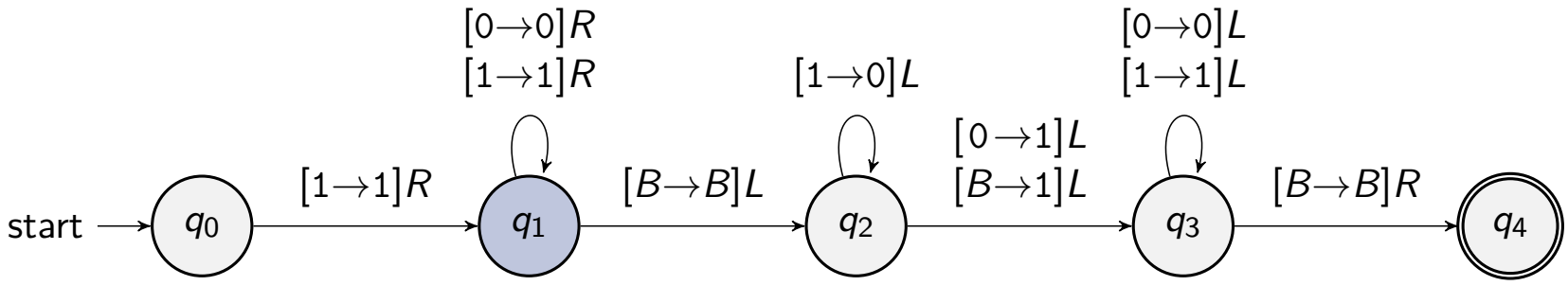

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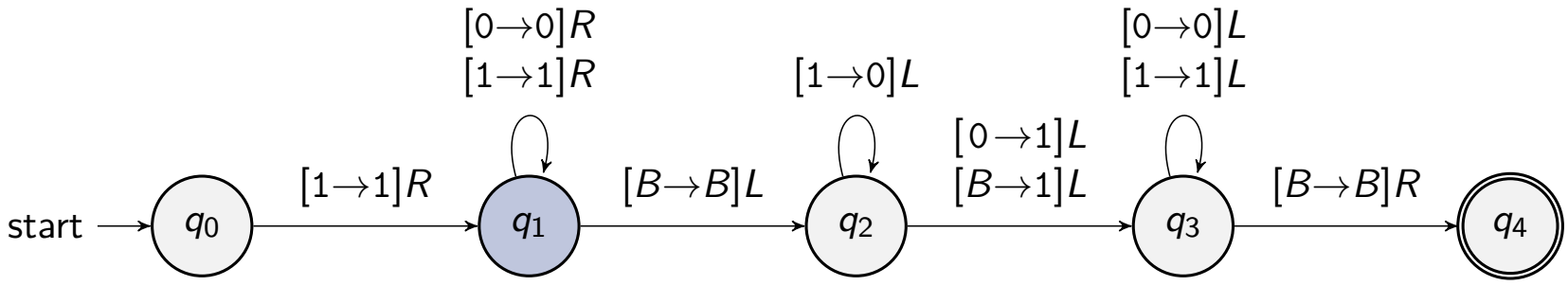

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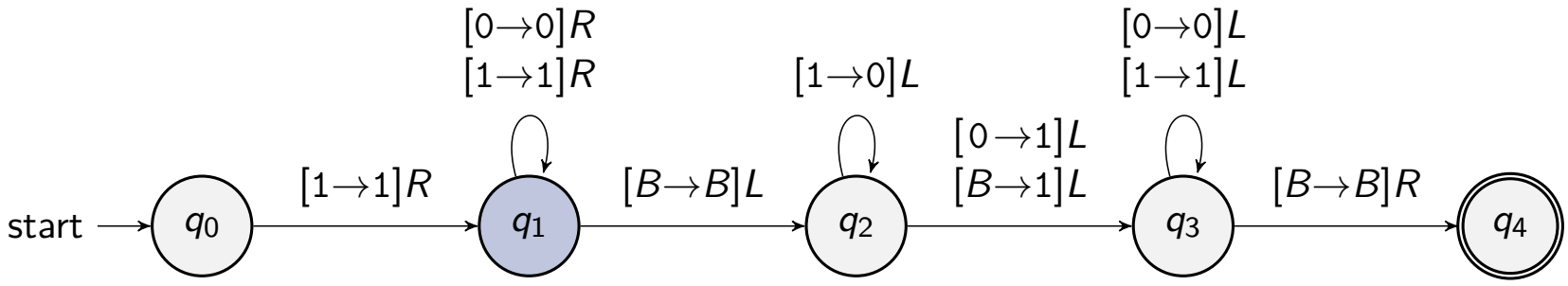

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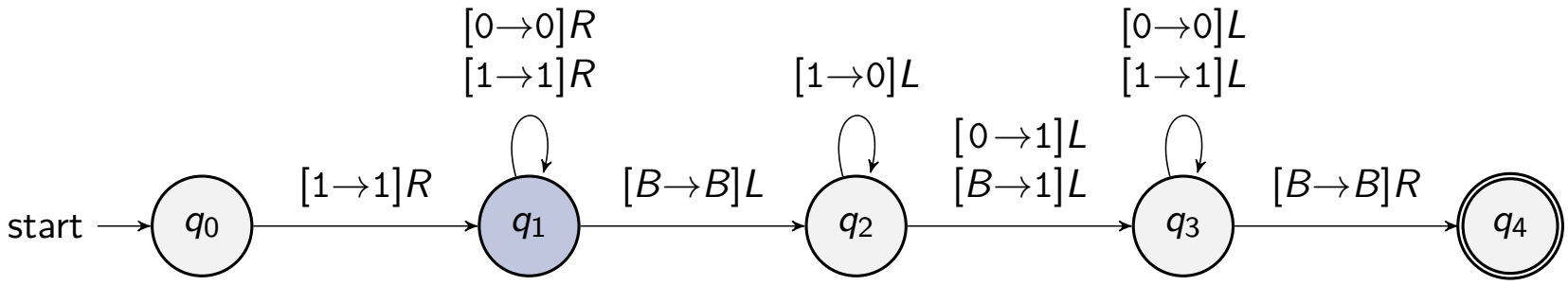

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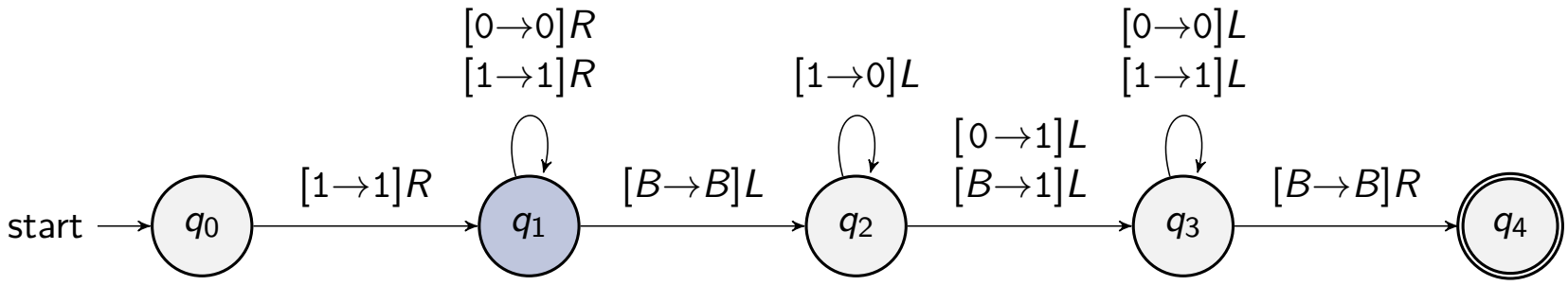

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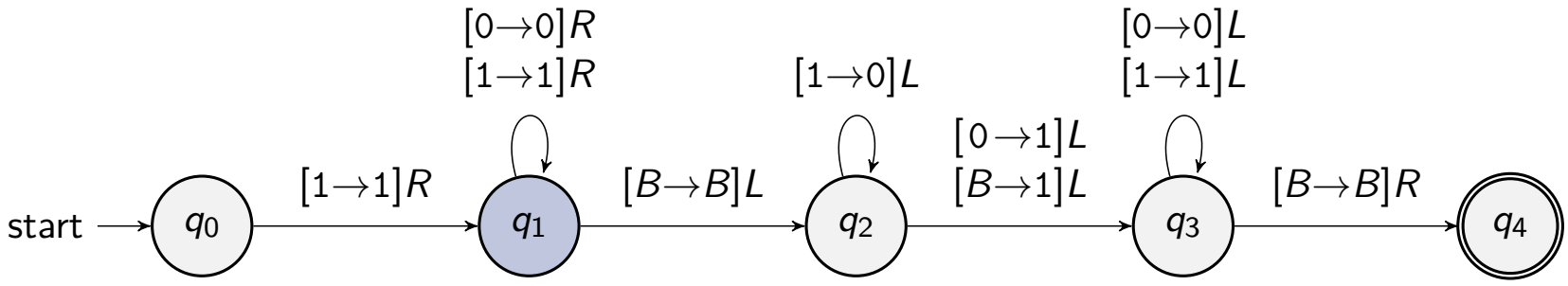

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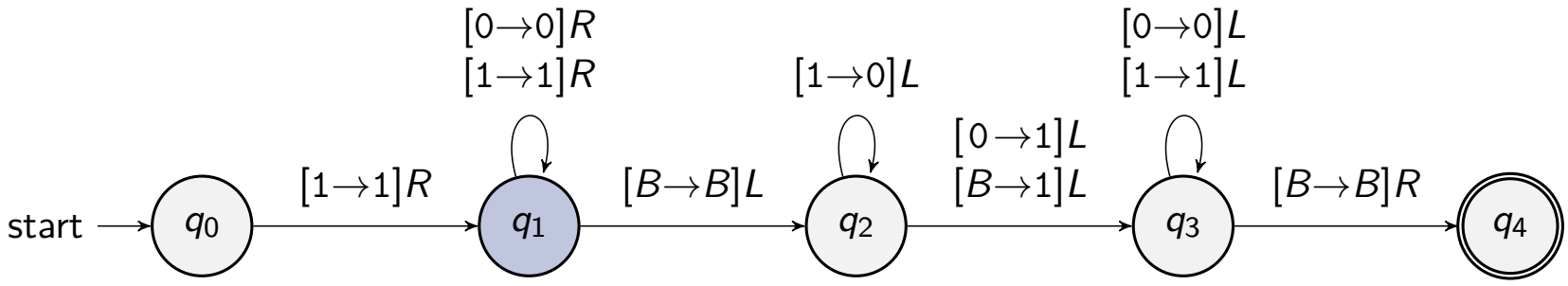

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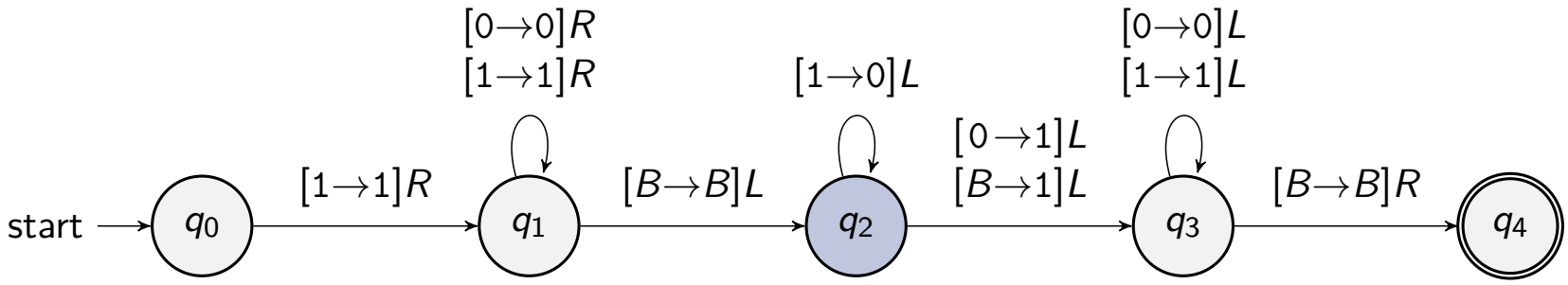

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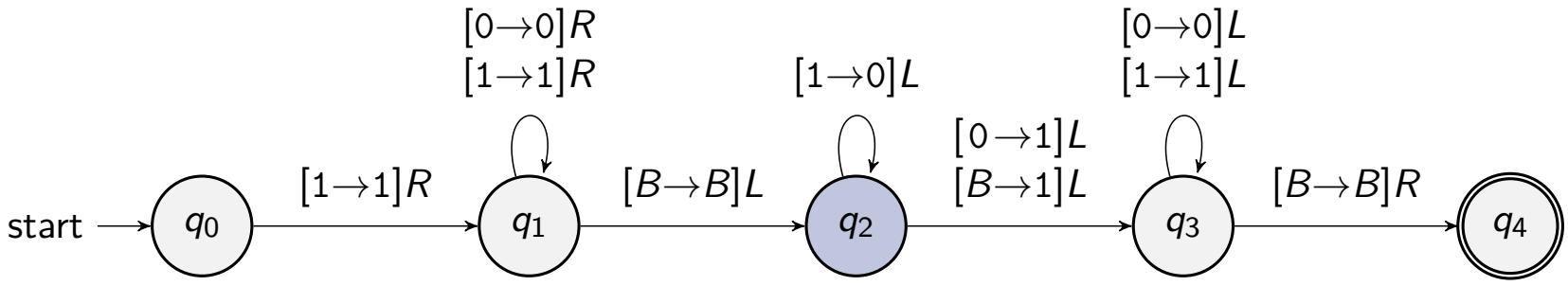
$\cdots$	B	1	0	1	0	1	1	1	0	B	$\cdots$



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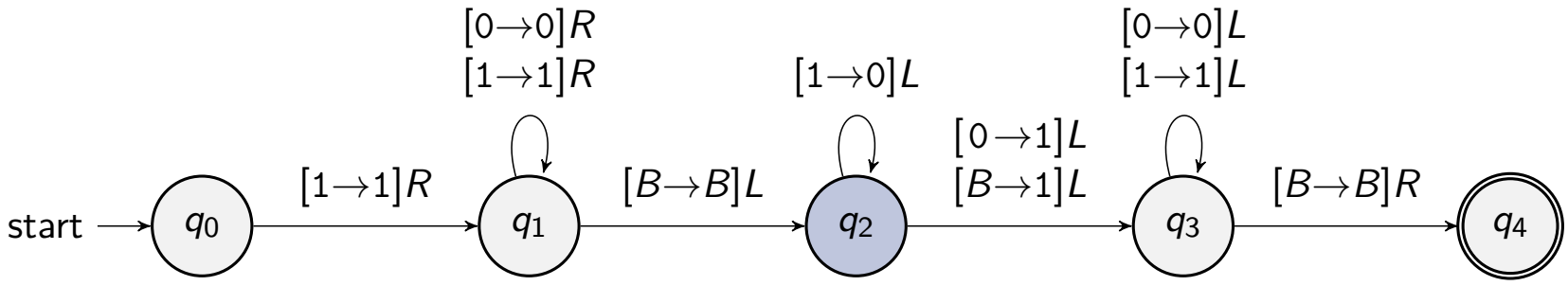

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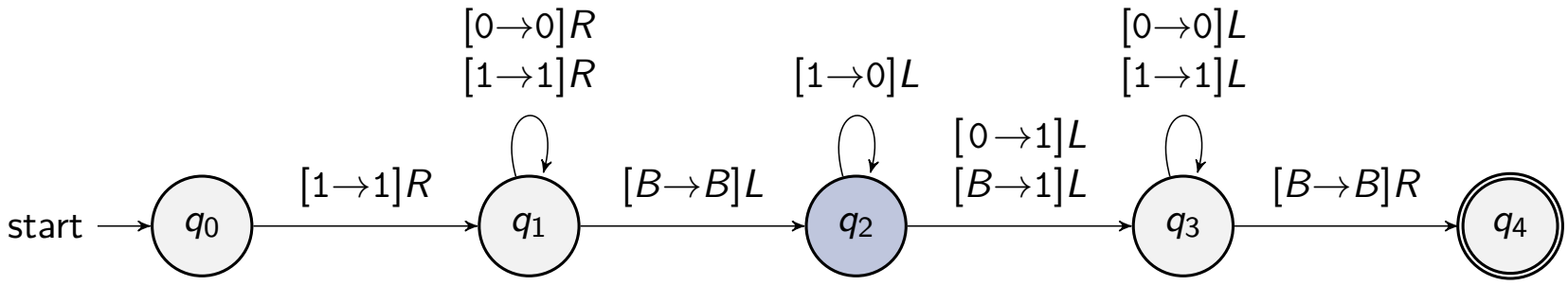

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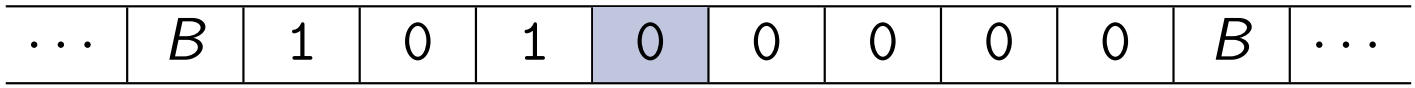

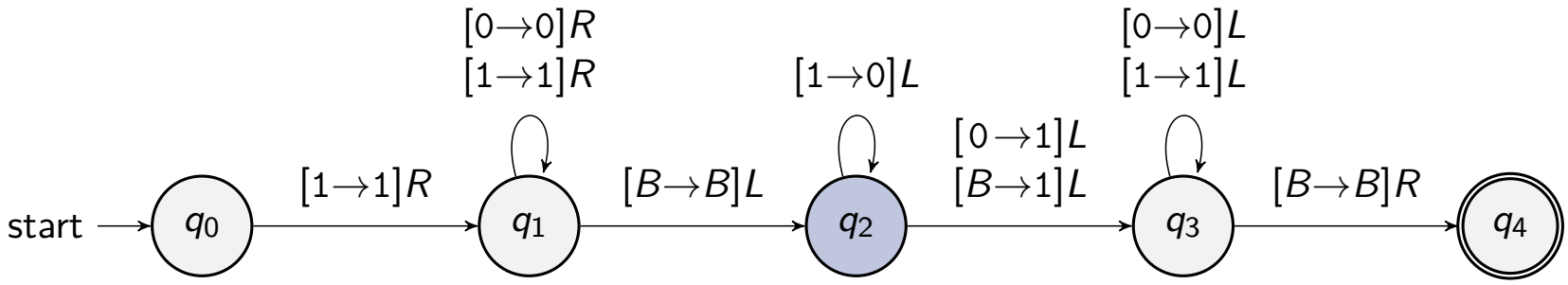

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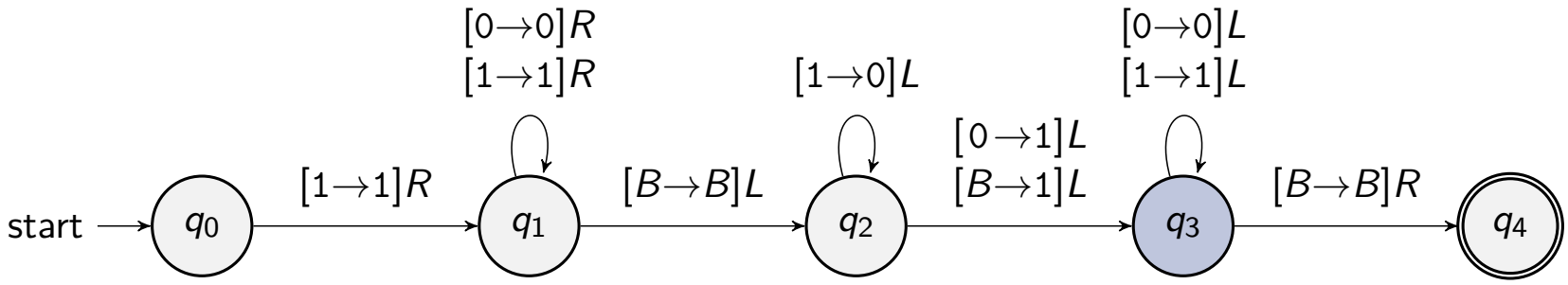

Go to the first input symbol.

Construct a **Turing machine** that **computes** the function:

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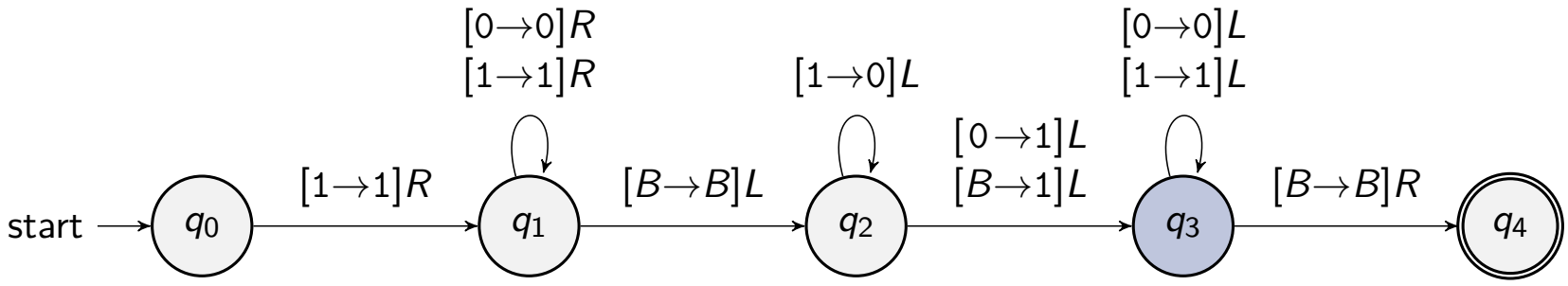
$\cdots$	B	1	0	1	1	0	0	0	0	B	$\cdots$



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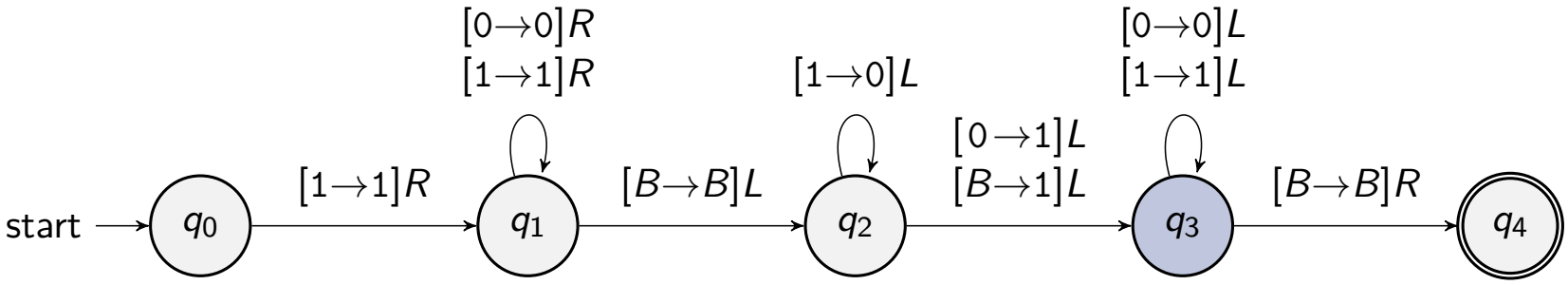

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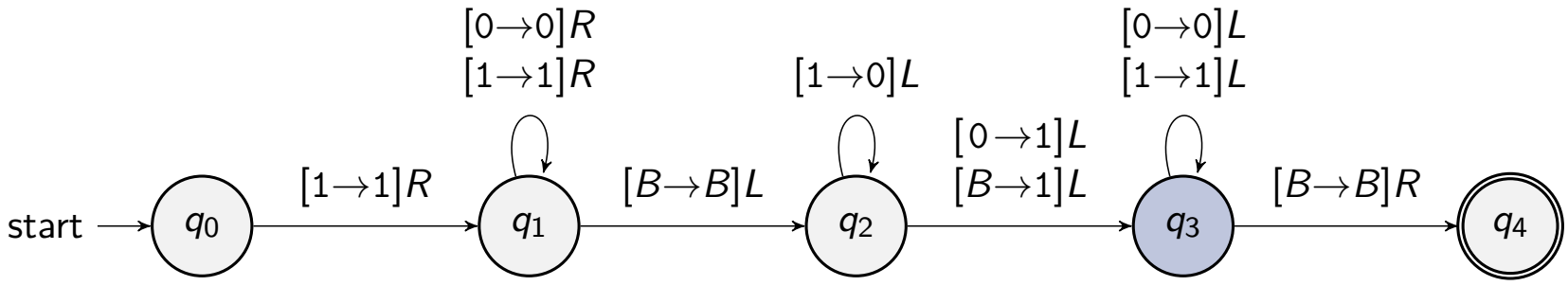

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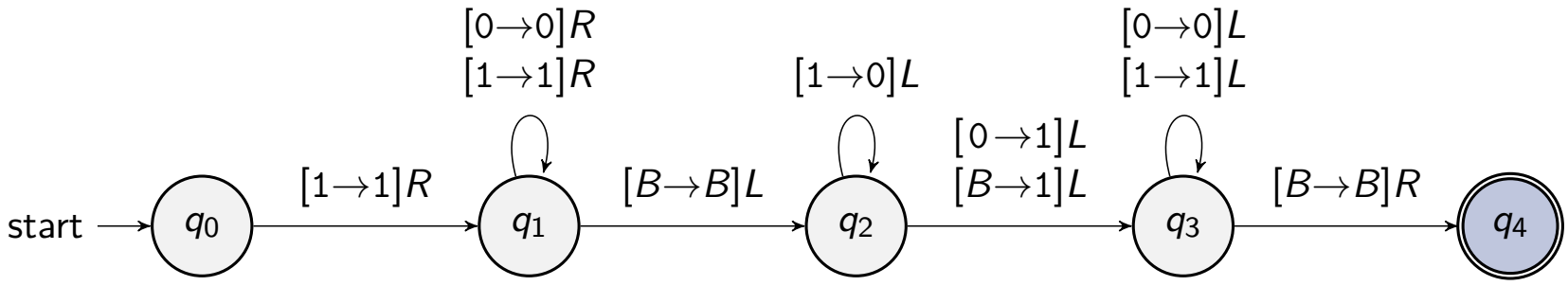

Computed! $f(10101111) = 10110000$